Multi-Microphone Speech Dereverberation using Eigen-decomposition

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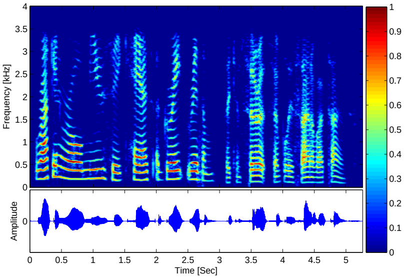
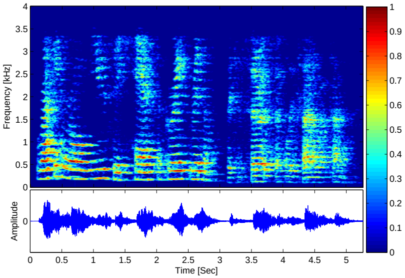
The Reverberation Phenomenon

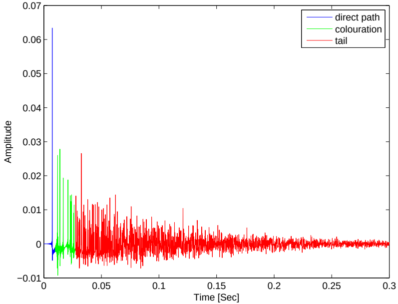


(a) Clean signal



(b) Reverberant signal ($T_{60} = 0.4s$)

The Room impulse Response (RIR)



The talk is based on:

- S. Gannot and M. Moonen, "Subspace methods for multi-microphone speech dereverberation," EURASIP J. Appl. Signal Process., vol. 2003, no. 1, pp. 1074-1090, 2003.
- S. Gannot, "Multi-Microphone Speech Dereverberation using Eigen-decomposition", to appear in "Speech Dereverberation", P.A. Naylor and N.D. Gaubich (Eds.), Springer, 2008.

Outline

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1. Problem Formulation
2. Preliminaries

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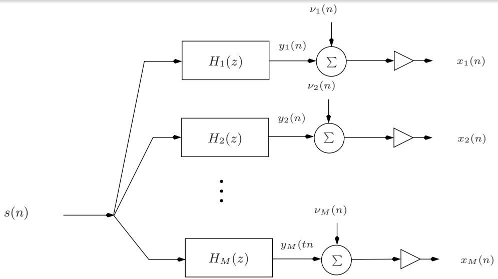
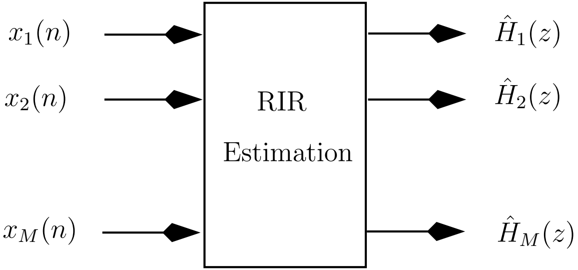
Problem Formulation



$$x_m(n) = y_m(n) + \nu_m(n) = \sum_{k=0}^{n_h} h_m(k) s(n-k) + \nu_m(n)$$

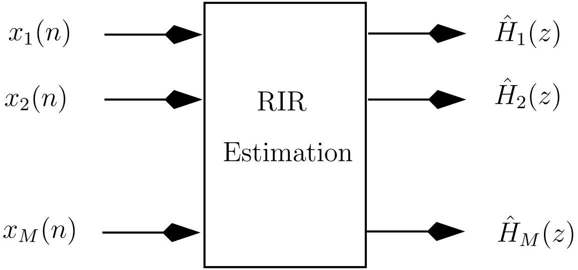
$$H_m(z) = \sum_{k=0}^{n_h} h_m(k) z^{-k}; \quad m = 1, 2, \ldots, M.$$

Goal



Use a Two Stage Approach

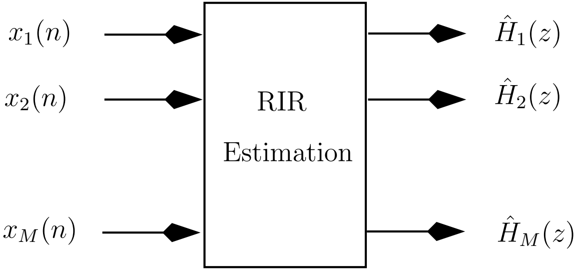
Goal



Use a Two Stage Approach

- Estimate the Acoustic Transfer Function (ATFs) $H_m(z)$.

Goal

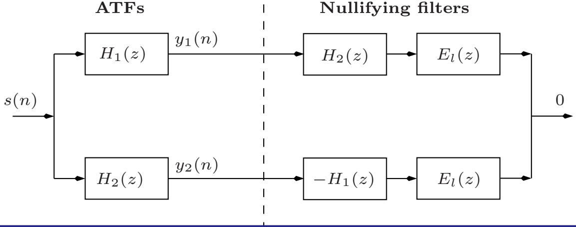


Use a Two Stage Approach

- Estimate the Acoustic Transfer Function (ATFs) $H_m(z)$.
- Use $\hat{H}_m(z);\ m = 1, \ldots, M$ to extract $s(n)$.

Two Microphone, Noiseless Case

$$y_1(n) = h_1(n) * s(n)$$
$$y_2(n) = h_2(n) * s(n)$$



Nullifying Filters

$$[y_2(n) * h_1(n) - y_1(n) * h_2(n)] * e_\ell(n) = 0$$
$$\tilde{h}_{m,\ell}(n) = h_m(n) * e_\ell(n); \; m = 1, 2$$

Data Matrix

$$\mathbf{Y}_m^T = \begin{bmatrix} y_m(0) & 0 & \cdots & 0 \\ y_m(1) & y_m(0) & & \vdots \\ \vdots & y_m(1) & \ddots & 0 \\ y_m(\hat{n}_h - 1) & \vdots & \ddots & y_m(0) \\ y_m(\hat{n}_h) & y_m(\hat{n}_h - 1) & & y_m(1) \\ \vdots & y_m(\hat{n}_h) & \ddots & \vdots \\ y_m(N) & \vdots & \ddots & y_m(\hat{n}_h - 1) \\ 0 & y_m(N) & \ddots & y_m(\hat{n}_h) \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & 0 & y_m(N) \end{bmatrix}$$

Filtered Room Impulse Responses (RIRs)

Define:

$$\tilde{\mathbf{h}}_{m,\ell}^T = \left[\, \tilde{h}_{m,\ell}(0) \;\; \tilde{h}_{m,\ell}(1) \;\; \ldots \;\; \tilde{h}_{m,\ell}(\hat{n}_h) \,\right];\;\; m = 1, 2$$

Concatenate:

$$\tilde{\mathbf{h}}_\ell = \begin{bmatrix} \tilde{\mathbf{h}}_{1,\ell} \\ \tilde{\mathbf{h}}_{2,\ell} \end{bmatrix}; \quad \mathbf{Y} = \begin{bmatrix} \mathbf{Y}_2 \\ -\mathbf{Y}_1 \end{bmatrix}$$

Nullifying Filters:

$$\mathbf{Y}^T \tilde{\mathbf{h}}_\ell = 0;\; \forall \ell.$$

Therefore:

$$\tilde{\mathbf{h}}_\ell \mathbf{Y} \mathbf{Y}^T \tilde{\mathbf{h}}_\ell = 0 \Rightarrow \tilde{\mathbf{h}}_\ell \hat{\mathbf{R}}_y \tilde{\mathbf{h}}_\ell = 0;\; \forall \ell$$

Null Subspace

Eigenvalue (or Singular Value) Decomposition

$$\lambda_\ell = 0 \;\; \ell = 0, 1, \ldots, \hat{n}_h - n_h$$
$$\lambda_\ell > 0 \text{ otherwise}$$

Null Subspace

Eigenvalue (or Singular Value) Decomposition

$$\lambda_\ell = 0 \quad \ell = 0, 1, \ldots, \hat{n}_h - n_h$$
$$\lambda_\ell > 0 \text{ otherwise}$$

Null Subspace Vectors

$$\mathbf{V} = \begin{bmatrix} \mathbf{v}_0 & \mathbf{v}_1 & \cdots & \mathbf{v}_{\hat{n}_h - n_h} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{h}}_{1,0} & \tilde{\mathbf{h}}_{1,1} & \cdots & \tilde{\mathbf{h}}_{1,\hat{n}_h - n_h} \\ \tilde{\mathbf{h}}_{2,0} & \tilde{\mathbf{h}}_{2,1} & \cdots & \tilde{\mathbf{h}}_{2,\hat{n}_h - n_h} \end{bmatrix}$$

Over-Estimated Room Impulse Responses

Acoustical Transfer Functions

For $\ell = 0, 1, \ldots, \hat{n}_h - n_h, \; m = 1, 2$:

$$\tilde{\mathbf{h}}_\ell \Leftrightarrow \tilde{H}_{m,\ell}(z)$$
$$\tilde{H}_{m,\ell}(z) = H_m(z) E_\ell(z)$$

Over-Estimated Room Impulse Responses

Acoustical Transfer Functions

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Fundamental Lemma

Over-Estimated Room Impulse Responses

Acoustical Transfer Functions

For $\ell = 0, 1, \ldots, \hat{n}_h - n_h,\ m = 1, 2$:

$$\tilde{\mathbf{h}}_\ell \Leftrightarrow \tilde{H}_{m,\ell}(z)$$
$$\tilde{H}_{m,\ell}(z) = H_m(z) E_\ell(z)$$

Fundamental Lemma

- For $m = 1, 2, \ldots, M$:
  $\tilde{H}_{m,\ell}(z)$ have $\hat{n}_h - n_h$ common roots $\Rightarrow E_l(z)$.

Over-Estimated Room Impulse Responses

Acoustical Transfer Functions

For $\ell = 0, 1, \ldots, \hat{n}_h - n_h,\; m = 1, 2$:

$$\tilde{\mathbf{h}}_\ell \Leftrightarrow \tilde{H}_{m,\ell}(z)$$
$$\tilde{H}_{m,\ell}(z) = H_m(z) E_\ell(z)$$

Fundamental Lemma

- For $m = 1, 2, \ldots, M$:
  $\tilde{H}_{m,\ell}(z)$ have $\hat{n}_h - n_h$ common roots $\Rightarrow E_l(z)$.
- For $\ell = 0, 1, \ldots, \hat{n}_h - n_h$:
  $\tilde{H}_{m,\ell}(z)$ have n_h common roots $\Rightarrow H_m(z)$.

RIR Estimation - Algorithm Derivation

Filtering (Silvester) Matrix:

$$\mathbf{H}_m = \underbrace{\begin{bmatrix} h_m(0) & 0 & 0 & \cdots & 0 \\ h_m(1) & h_m(0) & 0 & \cdots & 0 \\ \vdots & h_m(1) & \ddots & & \vdots \\ h_m(n_h) & \vdots & \ddots & \ddots & 0 \\ 0 & h_m(n_h) & & \ddots & h_m(0) \\ \vdots & 0 & & & h_m(1) \\ & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & h_m(n_h) \end{bmatrix}}_{\hat{n}_h - n_h + 1}$$

Over-Estimated Room Impulse Responses

Matrix Form

Define:

$$\mathbf{e}_\ell^T = \left[\, e_\ell(0)\;\; e_\ell(1)\;\; \ldots\;\; e_\ell(\hat{n}_h - n_h) \,\right]$$

Extraneous Filters:

$$\mathbf{E} = \left[\, \mathbf{e}_0\;\; \mathbf{e}_1\;\; \cdots\;\; \mathbf{e}_{\hat{n}_h - n_h} \,\right].$$

Null Subspace Vectors (Over-estimated RIRs):

$$\mathbf{V} = \begin{bmatrix} \tilde{\mathbf{h}}_{1,0} & \tilde{\mathbf{h}}_{1,1} & \cdots & \tilde{\mathbf{h}}_{1,\hat{n}_h - n_h} \\ \tilde{\mathbf{h}}_{2,0} & \tilde{\mathbf{h}}_{2,1} & \cdots & \tilde{\mathbf{h}}_{2,\hat{n}_h - n_h} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} \mathbf{E} \triangleq \mathbf{H}\mathbf{E}$$

Define $\mathbf{E}^i \triangleq \text{inv}(\mathbf{E}) = \left[\, \mathbf{e}_0^i\;\; \mathbf{e}_1^i\;\; \cdots\;\; \mathbf{e}_{\hat{n}_h - n_h}^i \,\right]$
Then:

$$\mathbf{H} = \mathbf{V}\mathbf{E}^i$$

RIR Extraction

Exploiting the Silvester Structure

$$\underbrace{\begin{bmatrix} \mathbf{V} & \mathbf{O} & \cdots & \cdots & \cdots & \mathbf{O} & -\mathbf{S}^{(0)} \\ \mathbf{O} & \mathbf{V} & \mathbf{O} & \cdots & \cdots & \mathbf{O} & -\mathbf{S}^{(1)} \\ \vdots & \mathbf{O} & \ddots & & & \vdots & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \mathbf{O} & \vdots \\ \mathbf{O} & \mathbf{O} & \cdots & \cdots & \mathbf{O} & \mathbf{V} & -\mathbf{S}^{(\hat{n}_h - n_h)} \end{bmatrix}}_{\tilde{\mathbf{V}}} \underbrace{\begin{bmatrix} \mathbf{e}^i_0 \\ \mathbf{e}^i_1 \\ \vdots \\ \mathbf{e}^i_{\hat{n}_h - n_h} \\ \mathbf{h}_1 \\ \mathbf{h}_2 \end{bmatrix}}_{\boldsymbol{\theta}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{\mathbf{0}}$$

$\mathbf{O}$ - all-zeros matrix

$\mathbf{S}^{(\ell)}$ - shift by ℓ matrix $(\ell = 0, 1, \ldots, \hat{n}_h - n_h)$

Algorithm Summary

RIR Estimation - Basic Case

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- $\tilde{\mathbf{V}}\,\boldsymbol{\theta} = \mathbf{0}$

Algorithm Summary

RIR Estimation - Basic Case

- $\tilde{\mathbf{V}}\,\theta = \mathbf{0}$
- Find eigenvector of $\tilde{\mathbf{V}}$ corresponding to eigenvalue 0

Algorithm Summary

RIR Estimation - Basic Case

- $\tilde{\mathbf{V}}\,\theta = \mathbf{0}$
- Find eigenvector of $\tilde{\mathbf{V}}$ corresponding to eigenvalue 0
- Extract $\mathbf{h}_1, \mathbf{h}_2$ from the eigenvector

Extensions

- Two Microphone Noisy Case

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- Two Microphone Noisy Case
    - White Noise Case

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- Two Microphone Noisy Case
  - White Noise Case
  - Colored Noise Case

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- Two Microphone Noisy Case
  - White Noise Case
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- Multi-Microphone Case ($M > 2$)

Extensions

- Two Microphone Noisy Case
  - White Noise Case
  - Colored Noise Case
- Multi-Microphone Case ($M > 2$)
- Partial Knowledge of the Null Subspace

Two Microphone Noisy Case

$$\mathbf{X} = \mathbf{Y} + \boldsymbol{\Upsilon},$$

$\mathbf{X}$ - noisy signal data matrix
$\boldsymbol{\Upsilon}$ - noise-only data matrix

$$\hat{\mathbf{R}}_x \approx \hat{\mathbf{R}}_y + \hat{\mathbf{R}}_\nu$$

$\hat{\mathbf{R}}_x = \frac{\mathbf{X}\mathbf{X}^T}{N+1}$ - noisy signal correlation matrix
$\hat{\mathbf{R}}_\nu = \frac{\boldsymbol{\Upsilon}\boldsymbol{\Upsilon}^T}{N+1}$ - noise-only signal correlation matrix

White Noise

$\hat{\mathbf{R}}_\nu \approx \sigma_\nu^2 \mathbf{I}$

RIR Estimation - White Noise

White Noise
$\hat{\mathbf{R}}_\nu \approx \sigma_\nu^2 \mathbf{I}$

RIR Estimation - White Noise

- **V** - eigenvectors corresponding to eigenvalue σ_ν^2 (remains intact)

White Noise

$\hat{\mathbf{R}}_\nu \approx \sigma_\nu^2 \mathbf{I}$

RIR Estimation - White Noise

- $\mathbf{V}$ - eigenvectors corresponding to eigenvalue σ_ν^2 (remains intact)
- $\tilde{\mathbf{V}}\,\boldsymbol{\theta} = \boldsymbol{\epsilon}$

White Noise

$\hat{\mathbf{R}}_\nu \approx \sigma_\nu^2 \mathbf{I}$

RIR Estimation - White Noise

- $\mathbf{V}$ - eigenvectors corresponding to eigenvalue σ_ν^2 (remains intact)
- $\tilde{\mathbf{V}}\,\boldsymbol{\theta} = \boldsymbol{\epsilon}$
- Find eigenvector of $\tilde{\mathbf{V}}$ corresponding to the smallest eigenvalue $\Rightarrow$Total Least Squares

White Noise

$\hat{\mathbf{R}}_\nu \approx \sigma_\nu^2 \mathbf{I}$

RIR Estimation - White Noise

- $\mathbf{V}$ - eigenvectors corresponding to eigenvalue σ_ν^2 (remains intact)
- $\tilde{\mathbf{V}}\,\boldsymbol{\theta} = \boldsymbol{\epsilon}$
- Find eigenvector of $\tilde{\mathbf{V}}$ corresponding to the smallest eigenvalue $\Rightarrow$Total Least Squares
- Extract $\mathbf{h}_1, \mathbf{h}_2$ from the eigenvector

Colored Noise

RIR Estimation - Colored Noise

Colored Noise

RIR Estimation - Colored Noise

- Calculate generalized EVD of $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_\nu$ (or generalized SVD of $\mathbf{X}$ and $\boldsymbol{\Upsilon}$)

Colored Noise

RIR Estimation - Colored Noise

- Calculate generalized EVD of $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_\nu$ (or generalized SVD of $\mathbf{X}$ and $\boldsymbol{\Upsilon}$)
- $\mathbf{V}$ - generalized eigenvectors corresponding to generalized eigenvalue 1

Colored Noise

RIR Estimation - Colored Noise

- Calculate generalized EVD of $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_\nu$ (or generalized SVD of $\mathbf{X}$ and $\boldsymbol{\Upsilon}$)
- $\mathbf{V}$ - generalized eigenvectors corresponding to generalized eigenvalue 1
- $\tilde{\mathbf{V}}\theta = \epsilon$

Colored Noise

RIR Estimation - Colored Noise

- Calculate generalized EVD of $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_\nu$ (or generalized SVD of $\mathbf{X}$ and $\boldsymbol{\Upsilon}$)
- $\mathbf{V}$ - generalized eigenvectors corresponding to generalized eigenvalue 1
- $\tilde{\mathbf{V}}\theta = \epsilon$
- Find eigenvector of $\tilde{\mathbf{V}}$ corresponding to the smallest eigenvalue $\Rightarrow$Total Least Squares

Colored Noise

RIR Estimation - Colored Noise

- Calculate generalized EVD of $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_\nu$ (or generalized SVD of $\mathbf{X}$ and $\boldsymbol{\Upsilon}$)
- $\mathbf{V}$ - generalized eigenvectors corresponding to generalized eigenvalue 1
- $\tilde{\mathbf{V}}\theta = \epsilon$
- Find eigenvector of $\tilde{\mathbf{V}}$ corresponding to the smallest eigenvalue $\Rightarrow$ Total Least Squares
- Extract $\mathbf{h}_1, \mathbf{h}_2$ from the eigenvector

Multi-Microphone Case ($M > 2$)

Pairing $\frac{M \times (M-1)}{2}$ channels:

$$[y_i(n) * h_j(n) - y_j(n) * h_i(n)] * e_l(n) = 0$$
$$i, j = 1, 2, \ldots, M; \;\; l = 0, 1, \ldots, \hat{n}_h - n_h$$

Construct an extended data matrix:

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_2 & \mathbf{X}_3 & \cdots & \mathbf{X}_M & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} \\ -\mathbf{X}_1 & \mathbf{0} & \cdots & & \mathbf{X}_3 & \cdots & \mathbf{X}_M & & \mathbf{0} \\ \mathbf{0} & -\mathbf{X}_1 & & & -\mathbf{X}_2 & & \mathbf{0} & & \vdots \\ \vdots & \mathbf{0} & \ddots & & & & \vdots & & \mathbf{0} \\ & \vdots & & \ddots & & & \mathbf{0} & & \mathbf{X}_M \\ \mathbf{0} & \mathbf{0} & \cdots & -\mathbf{X}_1 & \cdots & & -\mathbf{X}_2 & \cdots & -\mathbf{X}_{M-1} \end{bmatrix}$$

Algorithm

RIR Estimation - Multi-Microphone

Algorithm

RIR Estimation - Multi-Microphone

- Calculate generalized EVD of new $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_\nu$ (or generalized SVD of new $\mathbf{X}$ and $\boldsymbol{\Upsilon}$)

Algorithm

RIR Estimation - Multi-Microphone

- Calculate generalized EVD of new $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_\nu$ (or generalized SVD of new $\mathbf{X}$ and $\boldsymbol{\Upsilon}$)
- $\underline{\mathbf{V}}$ - new null subspace

Algorithm

RIR Estimation - Multi-Microphone

- Calculate generalized EVD of new $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_\nu$ (or generalized SVD of new $\mathbf{X}$ and $\boldsymbol{\Upsilon}$)
- $\underline{\mathbf{V}}$ - new null subspace
- $\underline{\tilde{\mathbf{V}}}\,\underline{\boldsymbol{\theta}} = \epsilon$, where:
  $\underline{\boldsymbol{\theta}}^T = \left[(\mathbf{e}_0^i)^T \; (\mathbf{e}_1^i)^T \; \cdots \; (\mathbf{e}_{\hat{n}_h - n_h}^i)^T \; \mathbf{h}_1^T \; \mathbf{h}_2^T \; \ldots \; \mathbf{h}_M^T \right]$

Algorithm

RIR Estimation - Multi-Microphone

- Calculate generalized EVD of new $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_\nu$ (or generalized SVD of new $\mathbf{X}$ and $\boldsymbol{\Upsilon}$)
- $\underline{\mathbf{V}}$ - new null subspace
- $\underline{\tilde{\mathbf{V}}}\,\underline{\boldsymbol{\theta}} = \epsilon$, where:
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- Find eigenvector of $\underline{\tilde{\mathbf{V}}}$ corresponding to the smallest eigenvalue $\Rightarrow$Total Least Squares

Algorithm

RIR Estimation - Multi-Microphone

- Calculate generalized EVD of new $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_\nu$ (or generalized SVD of new $\mathbf{X}$ and $\boldsymbol{\Upsilon}$)
- $\underline{\mathbf{V}}$ - new null subspace
- $\underline{\tilde{\mathbf{V}}}\,\underline{\boldsymbol{\theta}} = \epsilon$, where:
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- Find eigenvector of $\underline{\tilde{\mathbf{V}}}$ corresponding to the smallest eigenvalue $\Rightarrow$Total Least Squares
- Extract $\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_M$ from the eigenvector

Partial Knowledge of the Null Subspace

Augmented Null Subspace:

$$\bar{\mathbf{V}} = \begin{bmatrix} \mathbf{V} & \mathbf{0}^T & \mathbf{0}^T & & \mathbf{0}^T \\ \mathbf{0}^T & \mathbf{V} & \mathbf{0}^T & & \mathbf{0}^T \\ \vdots & & \ddots & & \mathbf{0}^T \\ & & & \ddots & \vdots \\ \mathbf{0}^T & & & & \mathbf{V} \end{bmatrix} = \bar{\mathbf{H}} \underbrace{\begin{bmatrix} \mathbf{E} & \mathbf{0}^T & \mathbf{0}^T & & \mathbf{0}^T \\ \mathbf{0}^T & \mathbf{E} & \mathbf{0}^T & & \mathbf{0}^T \\ \vdots & & \ddots & & \mathbf{0}^T \\ & & & \ddots & \vdots \\ \mathbf{0}^T & & & & \mathbf{E} \end{bmatrix}}_{L > \hat{n}_h - n_h + \hat{\ell}} \triangleq \bar{\mathbf{H}}\bar{\mathbf{E}}$$

$$\mathbf{E}^{Pi} = \mathrm{Pinv}\{\bar{\mathbf{E}}\} = \bar{\mathbf{E}}^T(\bar{\mathbf{E}}\bar{\mathbf{E}}^T)^{-1}$$

$$\Rightarrow \bar{\mathbf{V}}\mathbf{E}^{Pi} = \bar{\mathbf{H}}$$

Algorithm

RIR Estimation - Multi-Microphone

Algorithm

RIR Estimation - Multi-Microphone

- Calculate $\bar{\mathbf{V}}$ - augmented null subspace

Algorithm

RIR Estimation - Multi-Microphone

- Calculate $\bar{\mathbf{V}}$ - augmented null subspace
- $\bar{\tilde{\mathbf{V}}}\,\boldsymbol{\theta} = \boldsymbol{\epsilon}$

Algorithm

RIR Estimation - Multi-Microphone

- Calculate $\bar{\mathbf{V}}$ - augmented null subspace
- $\tilde{\bar{\mathbf{V}}}\,\boldsymbol{\theta} = \boldsymbol{\epsilon}$
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Algorithm

RIR Estimation - Multi-Microphone

- Calculate $\bar{\mathbf{V}}$ - augmented null subspace
- $\tilde{\bar{\mathbf{V}}}\,\boldsymbol{\theta} = \boldsymbol{\epsilon}$
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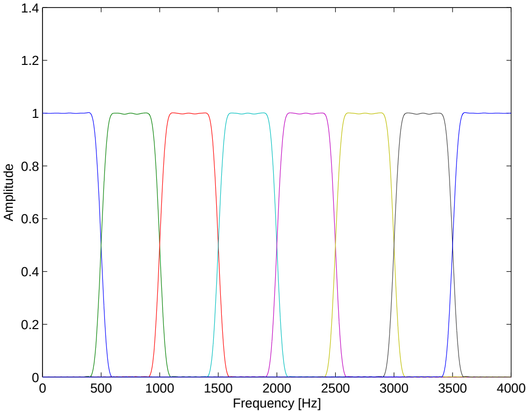
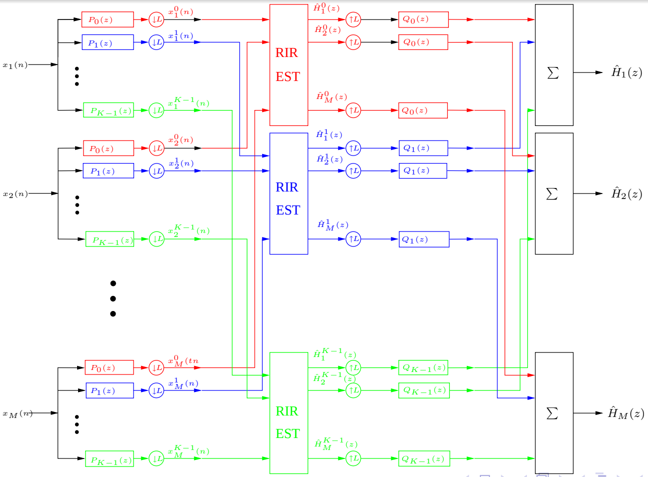
Subband Filters



RIR Estimation in Subbands



Signal Reconstruction (general)

$g_m(n)$; $m = 1, 2, \ldots, M$ - set of M equalizers.
Estimated speech signal:

$$\hat{s}(n) = \sum_{m=1}^{M} g_m(n) * x_m(n) =$$

$$\sum_{m=1}^{M} g_m(n) * h_m(n) * s(n) + \sum_{m=1}^{M} g_m(n) * \nu_m(n)$$

Equalization:

$$\sum_{m=1}^{M} g_m(n) * h_m(n) = \delta(n) \Leftrightarrow \sum_{m=1}^{M} G_m(z) H_m(z) = 1$$

Multi-channel Inverse Filter Theorem (MINT)

FIR Equalizers:

$$\mathbf{g}_m^T = \left[\, g_m(0)\ g_m(1)\ \ldots\ g_m(L_g) \,\right]$$

Causal equalization:

$$\underbrace{\left[\, \mathbf{H}_1\ \mathbf{H}_2\ \cdots\ \mathbf{H}_M \,\right]}_{\mathbf{H}} \underbrace{\begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \vdots \\ \mathbf{g}_M \end{bmatrix}}_{\mathbf{g}} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{\mathbf{d}}$$

$$\hat{\mathbf{g}} = \underset{\mathbf{g}}{\operatorname{argmin}} \left\| \mathbf{H}\mathbf{g} - \mathbf{d} \right\|^2 = \left(\mathbf{H}^T \mathbf{H} \right)^{-1} \mathbf{H}^T \mathbf{d}$$

Non-Causal Equalizers

Matched Beamformer (MBF)

$$G_m(z) = \frac{H_m^*(1/z^*)}{\sum_{m=1}^{M} H_m(z) H_m^*(1/z^*)} \Leftrightarrow G_m(e^{j\omega}) = \frac{H_m^*(e^{j\omega})}{\sum_{m=1}^{M} |H_m(e^{j\omega})|^2}.$$

Non-Causal Equalizers

Matched Beamformer (MBF)

$$G_m(z) = \frac{H_m^*(1/z^*)}{\sum_{m=1}^{M} H_m(z) H_m^*(1/z^*)} \Leftrightarrow G_m(e^{j\omega}) = \frac{H_m^*(e^{j\omega})}{\sum_{m=1}^{M} |H_m(e^{j\omega})|^2}.$$

Inverse Filter

$$G_m(z) = \frac{1}{H_m(z)} \Leftrightarrow G_m(e^{j\omega}) = \frac{1}{H_m(e^{j\omega})}$$

Experimental Study

Figures of Merit

- Inspection of the estimated RIR and ATF

Experimental Study

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- Inspection of the estimated RIR and ATF
- Comparison of the input speech signal, the reverberant signal, and the processed signal

Experimental Study

Figures of Merit

- Inspection of the estimated RIR and ATF
- Comparison of the input speech signal, the reverberant signal, and the processed signal
- Normalized Projection Misalignment (NPM)

$$\mathrm{NPM}\,[\mathrm{dB}] = 20\log_{10}\left(\frac{1}{\|h\|^2}\|h - \frac{(\mathbf{h}^T\hat{\mathbf{h}})^2\hat{\mathbf{h}}}{\|\hat{\mathbf{h}}\|^2}\|^2\right)$$

$$= 20\log_{10}\left(1 - \left(\frac{\mathbf{h}^T\hat{\mathbf{h}}}{\|\mathbf{h}\|\|\hat{\mathbf{h}}\|}\right)^2\right)$$

Full-band Version - Results

NPM vs. SNR

Scenario

$M = 2$, $n_h = 16$, $\hat{n}_h = 21$, $Fs = 8000$Hz, $T = 0.5$s, Discrete uniform distributed RIR coefficients, 50 "Monte Carlo" trials.

Full-band Version - Results

NPM vs. SNR

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$M = 2$, $n_h = 16$, $\hat{n}_h = 21$, $Fs = 8000$Hz, $T = 0.5$s, Discrete uniform distributed RIR coefficients, 50 "Monte Carlo" trials.

White Noise Input

SNR	15	20	25	30	35	40	45
NPM	-3.5	-8.6	-16.5	-28.0	-35.0	-44.0	-53

Full-band Version - Results

NPM vs. SNR

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$M = 2$, $n_h = 16$, $\hat{n}_h = 21$, $Fs = 8000$Hz, $T = 0.5$s, Discrete uniform distributed RIR coefficients, 50 “Monte Carlo” trials.

White Noise Input

SNR	15	20	25	30	35	40	45
NPM	-3.5	-8.6	-16.5	-28.0	-35.0	-44.0	-53

Speech Input

SNR	35	40	45	50	55	60	65
NPM	0.0	0.0	-2.0	-10.0	-11.0	-24.5	-38.0

Full-band Version - Results

NPM vs. filter order

Scenario

$M = 2$, SNR=50dB, $\hat{n}_h - n_h = 5$, $Fs = 8000$Hz, $T = 0.5$s, Gaussian distributed with decaying envelope RIR coefficients, 50 "Monte Carlo" trials.

Full-band Version - Results

NPM vs. filter order

Scenario

$M = 2$, SNR=50dB, $\hat{n}_h - n_h = 5$, $Fs = 8000\text{Hz}$, $T = 0.5\text{s}$, Gaussian distributed with decaying envelope RIR coefficients, 50 "Monte Carlo" trials.

White Noise Input

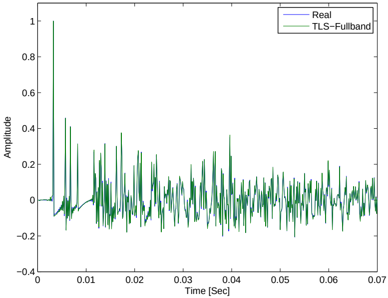
n_h	16	32	64	128	256
NPM	-60.0	-49.5	-33.0	-18.0	-0.5

Full-band Version - Results

Truncated Simulated RIR

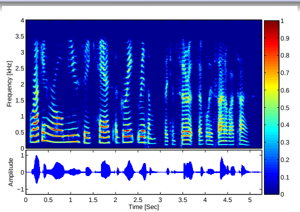
Scenario

$M = 2$, SNR=50dB, $\hat{n}_h - n_h = 5$, $Fs = 8000$Hz, $T = 0.5$s, $T_{60} = 0.7$s, RIR truncated to $n_h = 600$. NPM=-26dB.

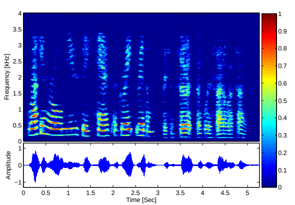


Full-band Version - Results

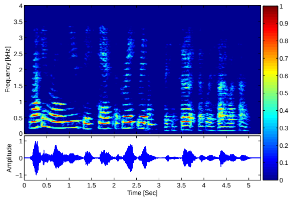
Sonograms



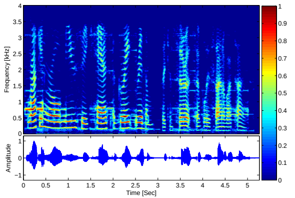
(a) Clean signal



(b) Reverberant signal (500 taps)



(a) Dereverberated signal (MINT)

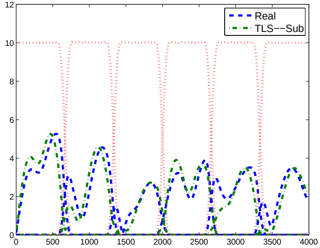


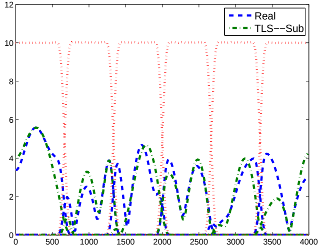
(b) Dereverberated signal (MBF)

Subband Version - Results

Scenario

$M = 2$, SNR=120dB, $n_h = 24$, 6 bands, $\hat{n}_h^k - n_h^k = 2$ per-band, T=4000, Gaussian distributed with decaying envelope RIR coefficients, white noise input, gain ambiguity compensated.





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  - Band overlap

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- Subband structures might be able to bring the prospective solution for the dereverberation problem