

Multi-Microphone Speech Dereverberation using Eigen-decomposition

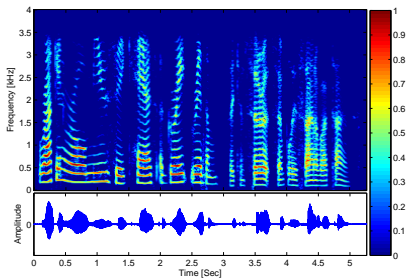
Sharon Gannot



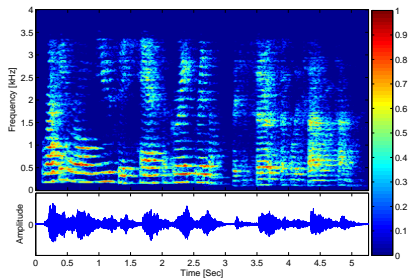
School of Electrical Engineering, Bar-Ilan University

Signal Processing and Systems (SP&S) Seminar, February 3rd, 2008

The Reverberation Phenomenon

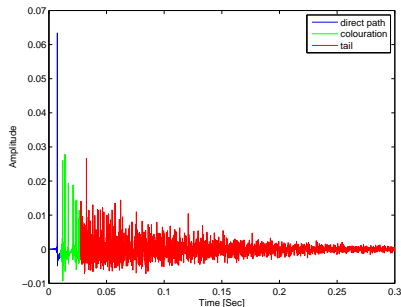


(a) Clean signal



(b) Reverberant signal ($T_{60} = 0.4s$)

The Room impulse Response (RIR)



The talk is based on:

- S. Gannot and M. Moonen, "Subspace methods for multi-microphone speech dereverberation," EURASIP J. Appl. Signal Process., vol. 2003, no. 1, pp. 1074-1090, 2003.
- S. Gannot, "Multi-Microphone Speech Dereverberation using Eigen-decomposition", to appear in "Speech Dereverberation", P.A. Naylor and N.D. Gaubich (Eds.), Springer, 2008.

- Problem Formulation
- Preliminaries
- RIR Estimation - Algorithm Derivation
- Extensions of the Basic Algorithm
- RIR Estimation in Subbands
- Signal Reconstruction
- Experimental Study
- Summary and Conclusions

Outline

1 Problem Formulation

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- 2 Preliminaries

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- 3 RIR Estimation - Algorithm Derivation

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- 3 RIR Estimation - Algorithm Derivation
- 4 Extensions of the Basic Algorithm

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- 4 Extensions of the Basic Algorithm
- 5 RIR Estimation in Subbands

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- 6 Signal Reconstruction

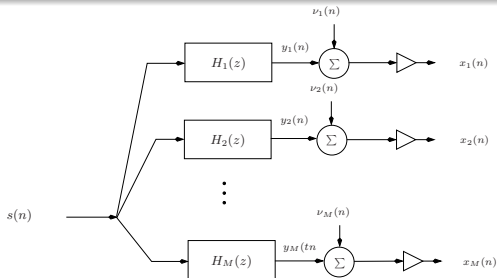
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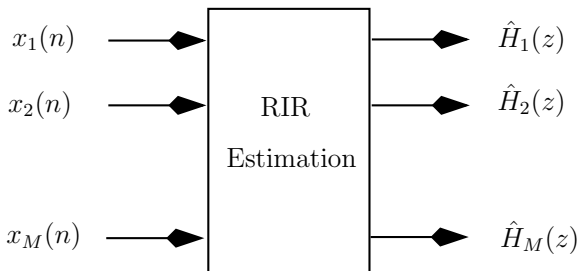
Problem Formulation



$$x_m(n) = y_m(n) + \nu_m(n) = \sum_{k=0}^{n_h} h_m(k)s(n-k) + \nu_m(n)$$

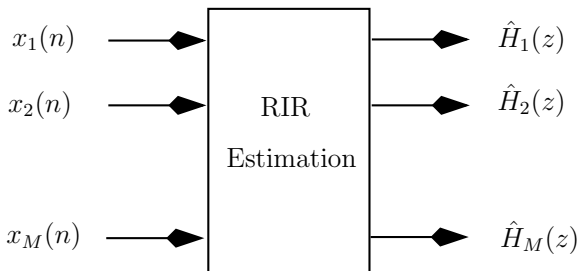
$$H_m(z) = \sum_{k=0}^{n_h} h_m(k)z^{-k}; \quad m = 1, 2, \dots, M.$$

Goal



Use a Two Stage Approach

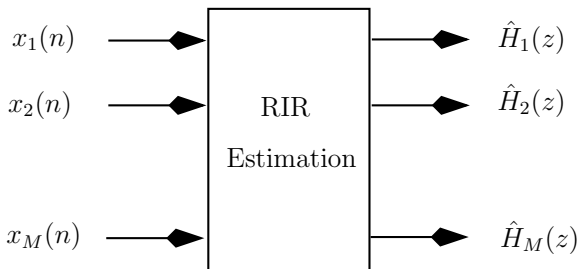
Goal



Use a Two Stage Approach

- Estimate the Acoustic Transfer Function (ATFs) $H_m(z)$.

Goal



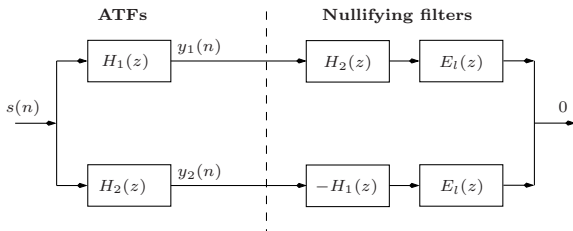
Use a Two Stage Approach

- Estimate the Acoustic Transfer Function (ATFs) $H_m(z)$.
- Use $\hat{H}_m(z)$; $m = 1, \dots, M$ to extract $s(n)$.

Two Microphone, Noiseless Case

$$y_1(n) = h_1(n) * s(n)$$

$$y_2(n) = h_2(n) * s(n)$$



Nullifying Filters

$$[y_2(n) * h_1(n) - y_1(n) * h_2(n)] * e_\ell(n) = 0$$

$$\tilde{h}_{m,\ell}(n) = h_m(n) * e_\ell(n); m = 1, 2$$

Data Matrix

$$\mathbf{Y}_m^T = \begin{bmatrix} y_m(0) & 0 & \cdots & 0 \\ y_m(1) & y_m(0) & & \vdots \\ \vdots & y_m(1) & \ddots & 0 \\ y_m(\hat{n}_h - 1) & \vdots & \ddots & y_m(0) \\ y_m(\hat{n}_h) & y_m(\hat{n}_h - 1) & & y_m(1) \\ \vdots & y_m(\hat{n}_h) & \ddots & \vdots \\ y_m(N) & \vdots & \ddots & y_m(\hat{n}_h - 1) \\ 0 & y_m(N) & \ddots & y_m(\hat{n}_h) \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & 0 & y_m(N) \end{bmatrix}$$

Filtered Room Impulse Responses (RIRs)

Define:

$$\tilde{\mathbf{h}}_{m,l}^T = [\tilde{h}_{m,l}(0) \ \tilde{h}_{m,l}(1) \ \dots \ \tilde{h}_{m,l}(\hat{n}_h)]; \quad m = 1, 2$$

Concatenate:

$$\tilde{\mathbf{h}}_\ell = \begin{bmatrix} \tilde{\mathbf{h}}_{1,\ell} \\ \tilde{\mathbf{h}}_{2,\ell} \end{bmatrix}; \quad \mathbf{Y} = \begin{bmatrix} \mathbf{Y}_2 \\ -\mathbf{Y}_1 \end{bmatrix}$$

Nullifying Filters:

$$\mathbf{Y}^T \tilde{\mathbf{h}}_\ell = 0; \quad \forall \ell.$$

Therefore:

$$\tilde{\mathbf{h}}_\ell \mathbf{Y} \mathbf{Y}^T \tilde{\mathbf{h}}_\ell = 0 \Rightarrow \tilde{\mathbf{h}}_\ell \hat{\mathbf{R}}_y \tilde{\mathbf{h}}_\ell = 0; \quad \forall \ell$$

Null Subspace

Eigenvalue (or Singular Value) Decomposition

$$\lambda_\ell = 0 \quad \ell = 0, 1, \dots, \hat{n}_h - n_h$$

$$\lambda_\ell > 0 \quad \text{otherwise}$$

Null Subspace

Eigenvalue (or Singular Value) Decomposition

$$\lambda_\ell = 0 \quad \ell = 0, 1, \dots, \hat{n}_h - n_h$$

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Null Subspace Vectors

$$\mathbf{V} = [\mathbf{v}_0 \ \mathbf{v}_1 \ \cdots \ \mathbf{v}_{\hat{n}_h - n_h}] = \begin{bmatrix} \tilde{\mathbf{h}}_{1,0} & \tilde{\mathbf{h}}_{1,1} & \cdots & \tilde{\mathbf{h}}_{1,\hat{n}_h - n_h} \\ \tilde{\mathbf{h}}_{2,0} & \tilde{\mathbf{h}}_{2,1} & \cdots & \tilde{\mathbf{h}}_{2,\hat{n}_h - n_h} \end{bmatrix}$$

Over-Estimated Room Impulse Responses

Acoustical Transfer Functions

For $\ell = 0, 1, \dots, \hat{n}_h - n_h$, $m = 1, 2$:

$$\begin{aligned}\tilde{\mathbf{h}}_\ell &\Leftrightarrow \tilde{H}_{m,\ell}(z) \\ \tilde{H}_{m,\ell}(z) &= H_m(z)E_\ell(z)\end{aligned}$$

Over-Estimated Room Impulse Responses

Acoustical Transfer Functions

For $\ell = 0, 1, \dots, \hat{n}_h - n_h$, $m = 1, 2$:

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Fundamental Lemma

Over-Estimated Room Impulse Responses

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Fundamental Lemma

- For $m = 1, 2, \dots, M$:
 $\tilde{H}_{m,\ell}(z)$ have $\hat{n}_h - n_h$ common roots $\Rightarrow E_l(z)$.

Over-Estimated Room Impulse Responses

Acoustical Transfer Functions

For $\ell = 0, 1, \dots, \hat{n}_h - n_h$, $m = 1, 2$:

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Fundamental Lemma

- For $m = 1, 2, \dots, M$:
 $\tilde{H}_{m,\ell}(z)$ have $\hat{n}_h - n_h$ common roots $\Rightarrow E_l(z)$.
- For $\ell = 0, 1, \dots, \hat{n}_h - n_h$:
 $\tilde{H}_{m,\ell}(z)$ have n_h common roots $\Rightarrow H_m(z)$.

RIR Estimation - Algorithm Derivation

Filtering (Silvester) Matrix:

$$\mathbf{H}_m = \underbrace{\begin{bmatrix} h_m(0) & 0 & 0 & \dots & 0 \\ h_m(1) & h_m(0) & 0 & \dots & 0 \\ \vdots & h_m(1) & \ddots & & \vdots \\ h_m(n_h) & \vdots & \ddots & \ddots & 0 \\ 0 & h_m(n_h) & & \ddots & h_m(0) \\ \vdots & 0 & & & h_m(1) \\ & \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & 0 & h_m(n_h) \end{bmatrix}}_{\hat{n}_h - n_h + 1}$$

Over-Estimated Room Impulse Responses

Matrix Form

Define:

$$\mathbf{e}_\ell^T = [e_\ell(0) \ e_\ell(1) \ \dots \ e_\ell(\hat{n}_h - n_h)]$$

Extraneous Filters:

$$\mathbf{E} = [\mathbf{e}_0 \ \mathbf{e}_1 \ \dots \ \mathbf{e}_{\hat{n}_h - n_h}]$$

Null Subspace Vectors (Over-estimated RIRs):

$$\mathbf{V} = \begin{bmatrix} \tilde{\mathbf{h}}_{1,0} & \tilde{\mathbf{h}}_{1,1} & \dots & \tilde{\mathbf{h}}_{1,\hat{n}_h - n_h} \\ \tilde{\mathbf{h}}_{2,0} & \tilde{\mathbf{h}}_{2,1} & \dots & \tilde{\mathbf{h}}_{2,\hat{n}_h - n_h} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} \mathbf{E} \triangleq \mathbf{H}\mathbf{E}$$

Define $\mathbf{E}^i \triangleq \text{inv}(\mathbf{E}) = [\mathbf{e}_0^i \ \mathbf{e}_1^i \ \dots \ \mathbf{e}_{\hat{n}_h - n_h}^i]$

Then:

$$\mathbf{H} = \mathbf{V}\mathbf{E}^i$$

RIR Extraction

Exploiting the Silvester Structure

$$\underbrace{\begin{bmatrix} \mathbf{V} & \mathbf{O} & \dots & \dots & \dots & \mathbf{O} & -\mathbf{S}^{(0)} \\ \mathbf{O} & \mathbf{V} & \mathbf{O} & \dots & \dots & \mathbf{O} & -\mathbf{S}^{(1)} \\ \vdots & \mathbf{O} & \ddots & & & \vdots & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & & & \ddots & \ddots & \vdots \\ \vdots & \vdots & & & & \mathbf{O} & \vdots \\ \mathbf{O} & \mathbf{O} & \dots & \dots & \mathbf{O} & \mathbf{V} & -\mathbf{S}^{(\hat{n}_h - n_h)} \end{bmatrix}}_{\tilde{\mathbf{V}}} \underbrace{\begin{bmatrix} \mathbf{e}_0^i \\ \mathbf{e}_1^i \\ \vdots \\ \mathbf{e}_{\hat{n}_h - n_h}^i \\ \mathbf{h}_1 \\ \mathbf{h}_2 \end{bmatrix}}_{\boldsymbol{\theta}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_0$$

\mathbf{O} - all-zeros matrix

$\mathbf{S}^{(\ell)}$ - shift by ℓ matrix ($\ell = 0, 1, \dots, \hat{n}_h - n_h$)

Algorithm Summary

RIR Estimation - Basic Case

Algorithm Summary

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- $\tilde{\mathbf{V}}\boldsymbol{\theta} = \mathbf{0}$

Algorithm Summary

RIR Estimation - Basic Case

- $\tilde{\mathbf{V}} \boldsymbol{\theta} = \mathbf{0}$
- Find eigenvector of $\tilde{\mathbf{V}}$ corresponding to eigenvalue 0

Algorithm Summary

RIR Estimation - Basic Case

- $\tilde{\mathbf{V}} \boldsymbol{\theta} = \mathbf{0}$
- Find eigenvector of $\tilde{\mathbf{V}}$ corresponding to eigenvalue 0
- Extract $\mathbf{h}_1, \mathbf{h}_2$ from the eigenvector

Extensions

- Two Microphone Noisy Case

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- Two Microphone Noisy Case
 - White Noise Case

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- Two Microphone Noisy Case
 - White Noise Case
 - Colored Noise Case

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- Two Microphone Noisy Case
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- Multi-Microphone Case ($M > 2$)

Extensions

- Two Microphone Noisy Case
 - White Noise Case
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- **Partial Knowledge of the Null Subspace**

Two Microphone Noisy Case

$$\mathbf{X} = \mathbf{Y} + \boldsymbol{\gamma},$$

\mathbf{X} - noisy signal data matrix

$\boldsymbol{\gamma}$ - noise-only data matrix

$$\hat{\mathbf{R}}_x \approx \hat{\mathbf{R}}_y + \hat{\mathbf{R}}_\nu$$

$\hat{\mathbf{R}}_x = \frac{\mathbf{X}\mathbf{X}^T}{N+1}$ - noisy signal correlation matrix

$\hat{\mathbf{R}}_\nu = \frac{\boldsymbol{\gamma}\boldsymbol{\gamma}^T}{N+1}$ - noise-only signal correlation matrix

White Noise

$$\hat{\mathbf{R}}_\nu \approx \sigma_\nu^2 \mathbf{I}$$

RIR Estimation - White Noise

White Noise

$$\hat{\mathbf{R}}_{\nu} \approx \sigma_{\nu}^2 \mathbf{I}$$

RIR Estimation - White Noise

- \mathbf{V} - eigenvectors corresponding to eigenvalue σ_{ν}^2
(remains intact)

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RIR Estimation - White Noise

- \mathbf{V} - eigenvectors corresponding to eigenvalue σ_ν^2
(remains intact)
- $\tilde{\mathbf{V}} \boldsymbol{\theta} = \boldsymbol{\epsilon}$

White Noise

$$\hat{\mathbf{R}}_\nu \approx \sigma_\nu^2 \mathbf{I}$$

RIR Estimation - White Noise

- \mathbf{V} - eigenvectors corresponding to eigenvalue σ_ν^2
(remains intact)
- $\tilde{\mathbf{V}} \boldsymbol{\theta} = \boldsymbol{\epsilon}$
- Find eigenvector of $\tilde{\mathbf{V}}$ corresponding to the smallest eigenvalue \Rightarrow Total Least Squares

White Noise

$$\hat{\mathbf{R}}_\nu \approx \sigma_\nu^2 \mathbf{I}$$

RIR Estimation - White Noise

- \mathbf{V} - eigenvectors corresponding to eigenvalue σ_ν^2
(remains intact)
- $\tilde{\mathbf{V}} \boldsymbol{\theta} = \boldsymbol{\epsilon}$
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- Extract $\mathbf{h}_1, \mathbf{h}_2$ from the eigenvector

Colored Noise

RIR Estimation - Colored Noise

Colored Noise

RIR Estimation - Colored Noise

- Calculate generalized EVD of $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_v$
(or generalized SVD of \mathbf{X} and \mathcal{Y})

Colored Noise

RIR Estimation - Colored Noise

- Calculate generalized EVD of $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_y$
(or generalized SVD of \mathbf{X} and \mathbf{Y})
- \mathbf{V} - generalized eigenvectors corresponding to generalized eigenvalue 1

Colored Noise

RIR Estimation - Colored Noise

- Calculate generalized EVD of $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_y$
(or generalized SVD of \mathbf{X} and \mathbf{Y})
- \mathbf{V} - generalized eigenvectors corresponding to generalized eigenvalue 1
- $\tilde{\mathbf{V}}\boldsymbol{\theta} = \boldsymbol{\epsilon}$

Colored Noise

RIR Estimation - Colored Noise

- Calculate generalized EVD of $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_y$
(or generalized SVD of \mathbf{X} and \mathbf{Y})
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Colored Noise

RIR Estimation - Colored Noise

- Calculate generalized EVD of $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_y$
(or generalized SVD of \mathbf{X} and \mathbf{Y})
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- Find eigenvector of $\tilde{\mathbf{V}}$ corresponding to the smallest eigenvalue \Rightarrow Total Least Squares
- Extract $\mathbf{h}_1, \mathbf{h}_2$ from the eigenvector

Multi-Microphone Case ($M > 2$)

Pairing $\frac{M \times (M-1)}{2}$ channels:

$$[y_i(n) * h_j(n) - y_j(n) * h_i(n)] * e_l(n) = 0$$

$$i, j = 1, 2, \dots, M; l = 0, 1, \dots, \hat{n}_h - n_h$$

Construct an extended data matrix:

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_2 & \mathbf{X}_3 & \cdots & \mathbf{X}_M & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} \\ -\mathbf{X}_1 & \mathbf{0} & \cdots & & \mathbf{X}_3 & \cdots & \mathbf{X}_M & & \mathbf{0} \\ \mathbf{0} & -\mathbf{X}_1 & & & -\mathbf{X}_2 & & \mathbf{0} & & \vdots \\ \vdots & \mathbf{0} & \ddots & & & & \vdots & & \mathbf{0} \\ & \vdots & & \ddots & & & \mathbf{0} & & \mathbf{X}_M \\ \mathbf{0} & \mathbf{0} & \cdots & -\mathbf{X}_1 & \cdots & -\mathbf{X}_2 & \cdots & -\mathbf{X}_{M-1} & \end{bmatrix}$$

Algorithm

RIR Estimation - Multi-Microphone

Algorithm

RIR Estimation - Multi-Microphone

- Calculate generalized EVD of new $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_v$
(or generalized SVD of new \mathbf{X} and \mathcal{Y})

Algorithm

RIR Estimation - Multi-Microphone

- Calculate generalized EVD of new $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_v$
(or generalized SVD of new \mathbf{X} and \mathcal{Y})
- \mathbf{V} - new null subspace

Algorithm

RIR Estimation - Multi-Microphone

- Calculate generalized EVD of new $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_v$
(or generalized SVD of new \mathbf{X} and \mathcal{Y})
- $\underline{\mathbf{V}}$ - new null subspace

- $\tilde{\mathbf{V}} \underline{\boldsymbol{\theta}} = \boldsymbol{\epsilon}$, where:

$$\underline{\boldsymbol{\theta}}^T = \left[(\mathbf{e}_0^i)^T \ (\mathbf{e}_1^i)^T \ \cdots \ (\mathbf{e}_{\hat{n}_h - n_h}^i)^T \ \mathbf{h}_1^T \ \mathbf{h}_2^T \ \cdots \ \mathbf{h}_M^T \right]$$

Algorithm

RIR Estimation - Multi-Microphone

- Calculate generalized EVD of new $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_v$
(or generalized SVD of new \mathbf{X} and \mathcal{Y})

- $\underline{\mathbf{V}}$ - new null subspace

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- Find eigenvector of $\underline{\tilde{\mathbf{V}}}$ corresponding to the smallest eigenvalue \Rightarrow Total Least Squares

Algorithm

RIR Estimation - Multi-Microphone

- Calculate generalized EVD of new $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_v$
(or generalized SVD of new \mathbf{X} and \mathcal{Y})
- $\underline{\mathbf{V}}$ - new null subspace
- $\underline{\tilde{\mathbf{V}}}\underline{\boldsymbol{\theta}} = \boldsymbol{\epsilon}$, where:

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- Find eigenvector of $\underline{\tilde{\mathbf{V}}}$ corresponding to the smallest eigenvalue \Rightarrow Total Least Squares
- Extract $\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_M$ from the eigenvector

Partial Knowledge of the Null Subspace

Augmented Null Subspace:

$$\bar{\mathbf{V}} = \begin{bmatrix} \mathbf{V} & \mathbf{0}^T & \mathbf{0}^T & \mathbf{0}^T \\ \mathbf{0}^T & \mathbf{V} & \mathbf{0}^T & \mathbf{0}^T \\ \vdots & & \ddots & \mathbf{0}^T \\ & & & \ddots \\ \mathbf{0}^T & & & \mathbf{V} \end{bmatrix} = \bar{\mathbf{H}} \underbrace{\begin{bmatrix} \mathbf{E} & \mathbf{0}^T & \mathbf{0}^T & \mathbf{0}^T \\ \mathbf{0}^T & \mathbf{E} & \mathbf{0}^T & \mathbf{0}^T \\ \vdots & & \ddots & \mathbf{0}^T \\ & & & \ddots \\ \mathbf{0}^T & & & \mathbf{E} \end{bmatrix}}_{L > \hat{n}_h - n_h + \hat{\ell}} \triangleq \bar{\mathbf{H}} \bar{\mathbf{E}}$$

$$\mathbf{E}^{Pi} = \text{Pinv}\{\bar{\mathbf{E}}\} = \bar{\mathbf{E}}^T (\bar{\mathbf{E}} \bar{\mathbf{E}}^T)^{-1}$$

$$\Rightarrow \bar{\mathbf{V}} \mathbf{E}^{Pi} = \bar{\mathbf{H}}$$

Algorithm

RIR Estimation - Multi-Microphone

Algorithm

RIR Estimation - Multi-Microphone

- Calculate $\bar{\mathbf{V}}$ - augmented null subspace

Algorithm

RIR Estimation - Multi-Microphone

- Calculate $\bar{\mathbf{V}}$ - augmented null subspace
- $\tilde{\mathbf{V}}\theta = \epsilon$

Algorithm

RIR Estimation - Multi-Microphone

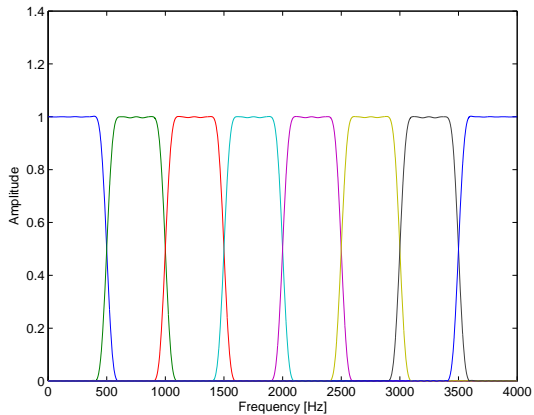
- Calculate $\bar{\mathbf{V}}$ - augmented null subspace
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Algorithm

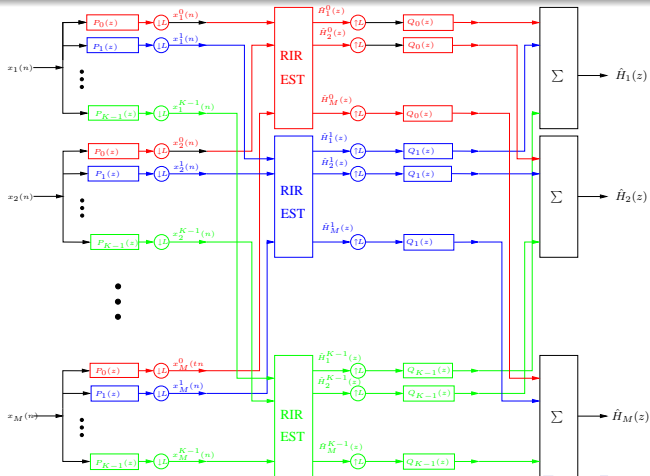
RIR Estimation - Multi-Microphone

- Calculate $\bar{\mathbf{V}}$ - augmented null subspace
- $\tilde{\mathbf{V}}\boldsymbol{\theta} = \boldsymbol{\epsilon}$
- Find eigenvector of $\tilde{\mathbf{V}}$ corresponding to the smallest eigenvalue \Rightarrow **Total Least Squares**
- **Extract $\mathbf{h}_1, \mathbf{h}_2$ from the eigenvector**

Subband Filters



RIR Estimation in Subbands



Signal Reconstruction (general)

$g_m(n)$; $m = 1, 2, \dots, M$ - set of M equalizers.

Estimated speech signal:

$$\hat{s}(n) = \sum_{m=1}^M g_m(n) * x_m(n) =$$

$$\sum_{m=1}^M g_m(n) * h_m(n) * s(n) + \sum_{m=1}^M g_m(n) * \nu_m(n)$$

Equalization:

$$\sum_{m=1}^M g_m(n) * h_m(n) = \delta(n) \Leftrightarrow \sum_{m=1}^M G_m(z)H_m(z) = 1$$

Multi-channel Inverse Filter Theorem (MINT)

FIR Equalizers:

$$\mathbf{g}_m^T = [g_m(0) \ g_m(1) \ \dots \ g_m(L_g)]$$

Causal equalization:

$$\underbrace{[\mathbf{H}_1 \ \mathbf{H}_2 \ \dots \ \mathbf{H}_M]}_{\mathbf{H}} \underbrace{\begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \vdots \\ \mathbf{g}_M \end{bmatrix}}_{\mathbf{g}} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{\mathbf{d}}$$

$$\hat{\mathbf{g}} = \underset{\mathbf{g}}{\operatorname{argmin}} \|\mathbf{H}\mathbf{g} - \mathbf{d}\|^2 = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{d}$$

Non-Causal Equalizers

Matched Beamformer (MBF)

$$G_m(z) = \frac{H_m^*(1/z^*)}{\sum_{m=1}^M H_m(z) H_m^*(1/z^*)} \Leftrightarrow G_m(e^{j\omega}) = \frac{H_m^*(e^{j\omega})}{\sum_{m=1}^M |H_m(e^{j\omega})|^2}$$

Non-Causal Equalizers

Matched Beamformer (MBF)

$$G_m(z) = \frac{H_m^*(1/z^*)}{\sum_{m=1}^M H_m(z) H_m^*(1/z^*)} \Leftrightarrow G_m(e^{j\omega}) = \frac{H_m^*(e^{j\omega})}{\sum_{m=1}^M |H_m(e^{j\omega})|^2}$$

Inverse Filter

$$G_m(z) = \frac{1}{H_m(z)} \Leftrightarrow G_m(e^{j\omega}) = \frac{1}{H_m(e^{j\omega})}$$

Experimental Study

Figures of Merit

- Inspection of the estimated RIR and ATF

Experimental Study

Figures of Merit

- Inspection of the estimated RIR and ATF
- Comparison of the input speech signal, the reverberant signal, and the processed signal

Experimental Study

Figures of Merit

- Inspection of the estimated RIR and ATF
- Comparison of the input speech signal, the reverberant signal, and the processed signal
- **Normalized Projection Misalignment (NPM)**

$$\begin{aligned} \text{NPM [dB]} &= 20 \log_{10} \left(\frac{1}{\|h\|^2} \left\| h - \frac{(\mathbf{h}^T \hat{\mathbf{h}})^2 \hat{\mathbf{h}}}{\|\hat{\mathbf{h}}\|^2} \right\|^2 \right) \\ &= 20 \log_{10} \left(1 - \left(\frac{\mathbf{h}^T \hat{\mathbf{h}}}{\|h\| \|\hat{\mathbf{h}}\|} \right)^2 \right) \end{aligned}$$

Full-band Version - Results

NPM vs. SNR

Scenario

$M = 2$, $n_h = 16$, $\hat{n}_h = 21$, $F_s = 8000\text{Hz}$, $T = 0.5\text{s}$, Discrete uniform distributed RIR coefficients, 50 “Monte Carlo” trials.

Full-band Version - Results

NPM vs. SNR

Scenario

$M = 2$, $n_h = 16$, $\hat{n}_h = 21$, $F_s = 8000\text{Hz}$, $T = 0.5\text{s}$, Discrete uniform distributed RIR coefficients, 50 "Monte Carlo" trials.

White Noise Input

SNR	15	20	25	30	35	40	45
NPM	-3.5	-8.6	-16.5	-28.0	-35.0	-44.0	-53

Full-band Version - Results

NPM vs. SNR

Scenario

$M = 2$, $n_h = 16$, $\hat{n}_h = 21$, $F_s = 8000\text{Hz}$, $T = 0.5\text{s}$, Discrete uniform distributed RIR coefficients, 50 "Monte Carlo" trials.

White Noise Input

SNR	15	20	25	30	35	40	45
NPM	-3.5	-8.6	-16.5	-28.0	-35.0	-44.0	-53

Speech Input

SNR	35	40	45	50	55	60	65
NPM	0.0	0.0	-2.0	-10.0	-11.0	-24.5	-38.0

Full-band Version - Results

NPM vs. filter order

Scenario

$M = 2$, SNR=50dB, $\hat{n}_h - n_h = 5$, $F_s = 8000\text{Hz}$, $T = 0.5\text{s}$,
Gaussian distributed with decaying envelope RIR coefficients, 50
“Monte Carlo” trials.

Full-band Version - Results

NPM vs. filter order

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Gaussian distributed with decaying envelope RIR coefficients, 50
“Monte Carlo” trials.

White Noise Input

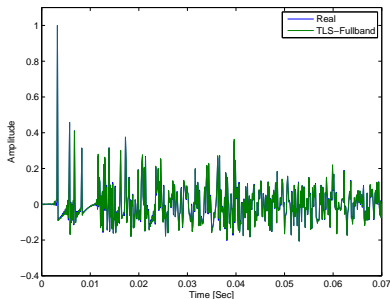
n_h	16	32	64	128	256
NPM	-60.0	-49.5	-33.0	-18.0	-0.5

Full-band Version - Results

Truncated Simulated RIR

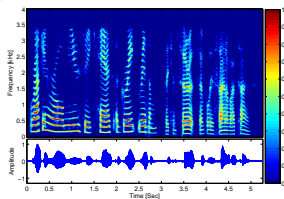
Scenario

$M = 2$, $\text{SNR} = 50\text{dB}$, $\hat{n}_h - n_h = 5$, $F_s = 8000\text{Hz}$, $T = 0.5\text{s}$,
 $T_{60} = 0.7\text{s}$, RIR truncated to $n_h = 600$. $\text{NPM} = -26\text{dB}$.

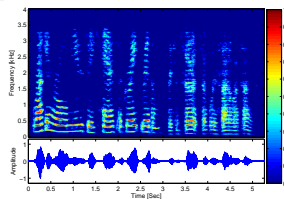


Full-band Version - Results

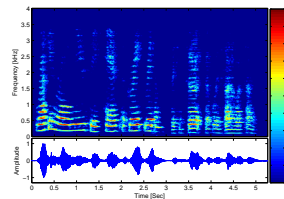
Sonograms



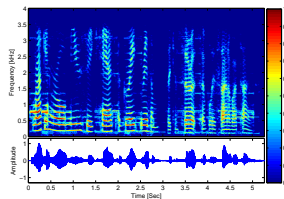
(a) Clean signal



(b) Reverberant signal (500 taps)



(a) Dereverberated signal (MINT)

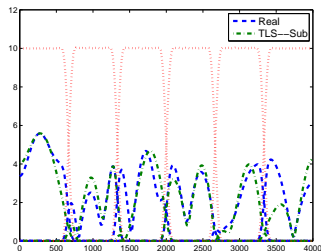
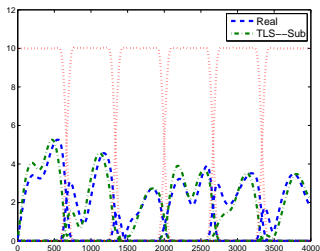


(b) Dereverberated signal (MBF)

Subband Version - Results

Scenario

$M = 2$, SNR=120dB, $n_h = 24$, 6 bands, $\hat{n}_h^k - n_h^k = 2$ per-band, $T=4000$, Gaussian distributed with decaying envelope RIR coefficients, white noise input, gain ambiguity compensated.



Limitations of the Proposed Methods

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 - Band overlap
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 - $\hat{n}_h \geq n_h$
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 - Band gaps
- **Gain Ambiguity**

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 - Subband method

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- **Subband structures might be able to bring the prospective solution for the dereverberation problem**