Introduction to distributed speech enhancement algorithms for ad hoc microphone arrays and wireless acoustic sensor networks

Part II: DANSE-based distributed speech enhancement in WASNs

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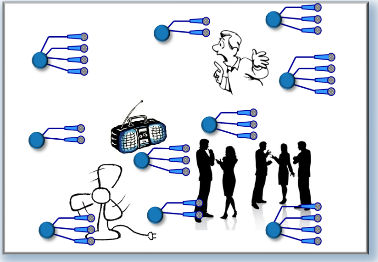
Outline

Ad hoc microphone arrays

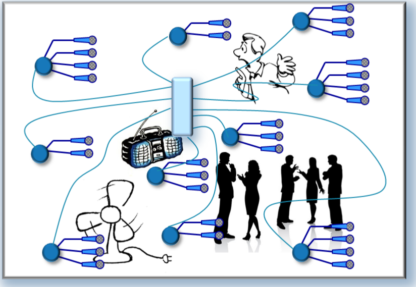
Ad hoc microphone arrays

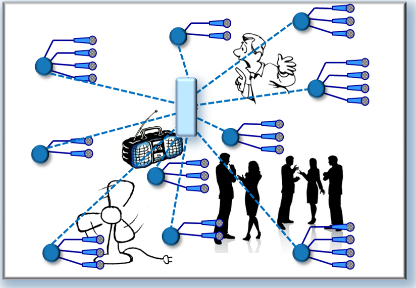
- No tedious calibration
- Improved spatial resolution and sound field sampling.
- High probability to find microphones close to a relevant sound source.
- Possibility to put (arrays of) microphones at strategic places



Wireless acoustic sensor networks (WASNs)

- Wired ad hoc arrays:
    - Tedious deployment
    - Unaesthetic
    - Not flexible (e.g., adding/removing/repositioning microphones)
    - Not suitable for wearable or mobile applications (e.g., hearing aids)
- Aim for *wireless* ad hoc microphone arrays.
- A.k.a. wireless acoustic sensor network (WASN)
  (due to similarities with wireless sensor networks)





Wireless acoustic sensor networks (WASNs)

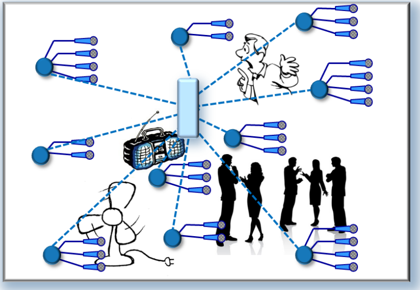
Possible applications:

- Cooperative hearing devices (e.g., binaural hearing aids)
- Hearing devices supported by external microphones or other audio devices
- Domotics, smart homes and ambient intelligence
- Surveillance
- ...

Wireless acoustic sensor networks (WASNs)

Challenges

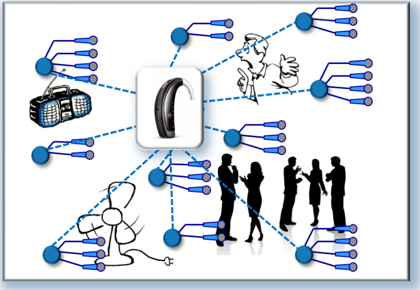
- Wireless link delay (e.g., in case of real-time constraints)
- Different sampling clocks (see also Part III)
- The 'data deluge' (see next slide)



WASNs and the data deluge

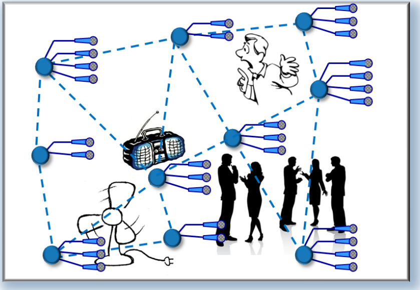
The 'data deluge' [Baraniuk, 2011]

- WASNs generate a massive amount of data:
  - Requires a large communication bandwidth
  - Sensor nodes consume a large amount of transmission energy
  - Requires high computing power at the receiver end (fusion center)
- =big problem, in particular when battery-powered (even in small-scale WASNs such as binaural hearing aids)



Distributed signal processing in WASNs

- Tackle the data deluge by physically shifting the signal processing to the microphone nodes themselves
- Goals:
  - Minimize data exchange
  - Distribute computational burden over all nodes
  - Let nodes cooperate in signal processing task(s)
- Algorithm design=challenging (e.g., no access to full correlation matrix)



Distributed signal processing

The field of distributed signal processing:

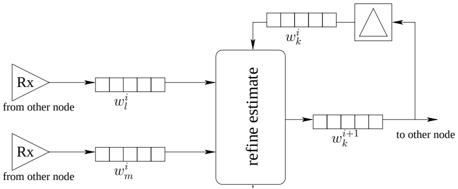
- Mainly driven by the concept of wireless sensor networks
- Theory and methods often build upon results from other fields, e.g.,
  - Parallel and distributed computing for multi-core processors
  - Modelling and control of multi-agent systems
  - Game theory
  - Graph theory
- Two fundamentally different approaches:
  1. Distributed parameter estimation techniques (DPE)
     (e.g., diffusion [Sayed et al., 2013], consensus [Olfati-Saber et al., 2007], gossip [Shah, 2009], ...)
  2. Distributed signal estimation techniques (DSE)
     (e.g., DANSE-family, distributed/cooperative beamforming, distributed/remote source coding, ...)

Distributed parameter estimation (DPE)

General script:

1. Extract initial parameter vector estimate from sensor observations
2. Repeat until convergence (or other stop criterion):
    - Share intermediate estimate with neighbors
    - Refine intermediate estimate using estimates from neighbors

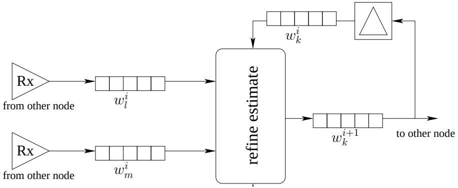
Note: target parameter vector is fixed over iterations, or varies only slowly



DPE for speech enhancement in WASNs

Collect L microphone signal samples at each node and iterate on L-dimensional vector until the estimate converges. Then collect L new samples, etc.

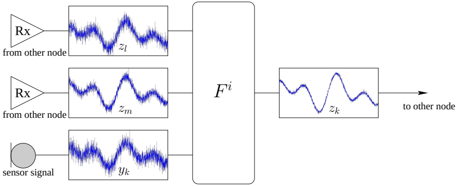
- ☺ DPE techniques usually have no network topology constraints
- ☹ Large communication cost: re-estimate and re-transmit same L samples many times (freeze time index until convergence)
- ☹ Communication cost depends on convergence speed (and hence also on network size)
- ☹ Not time-recursive: full reset between blocks



See, e.g., [Zeng and Hendriks, 2012, Heusdens et al., 2012]

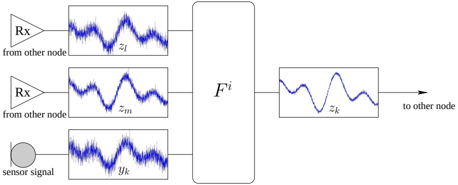
Distributed signal estimation (DSE)

- Avoid iterations over the signal sample estimates themselves
  $\Rightarrow$ In-network data flow and iterative process are uncoupled
- Instead: time-recursive iterative refinement of in-network fusion rules
- Assumption: spatial coherence of sensor signals is fixed over iterations (or varies slowly)



DSE for speech enhancement in WASNs?

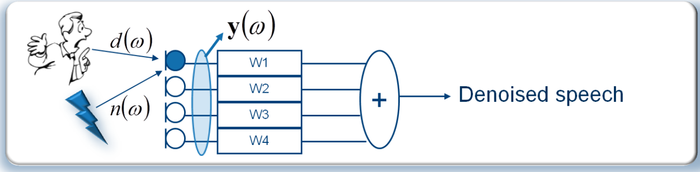
- No iterative refinement of sample estimates:
  ☺ Each block of samples is transmitted only once
  ☺ Fixed per-node communication cost, independent of convergence speed/network size
- ☹ Price to pay: specific order in data flow generally requires topology constraints (star, tree, fully-connected,...)



See, e.g., [Doclo et al., 2009, Bertrand and Moonen, 2009, Markovich-Golan et al., 2010, Markovich-Golan et al., 2013, Lawin-Ore and Doclo, 2011, Himawan et al., 2011, Hioka and Kleijn, 2011, Szurley et al., 2013]

Multi-channel Wiener filtering [Doclo and Moonen, 2002]

- Goal: estimate speech component at reference microphone
- Optimal filter-and-sum operation based on input statistics



$$\min_{\mathbf{w}} E\{| d_{ref} - \mathbf{w}^H \mathbf{y} |^2\}$$

$$\mathbf{w}(\omega) = \mathbf{R}_{yy}(\omega)^{-1} \mathbf{R}_{dd}(\omega)\, \mathbf{e}_{ref}$$

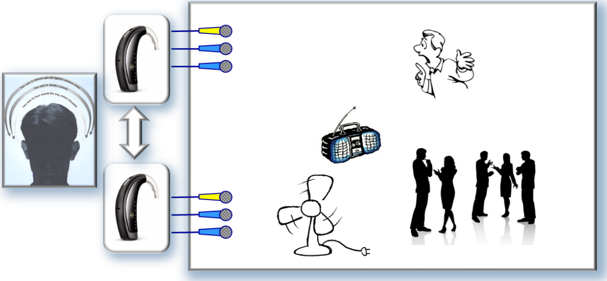
$$\mathbf{R}_{yy}(\omega) = E\{\mathbf{y}(\omega)\mathbf{y}(\omega)^H\}$$

Voice activity detection (VAD)

$$\mathbf{R}_{dd}(\omega) = \mathbf{R}_{yy}(\omega) - \mathbf{R}_{nn}(\omega)$$

Preliminary case study: binaural hearing aids

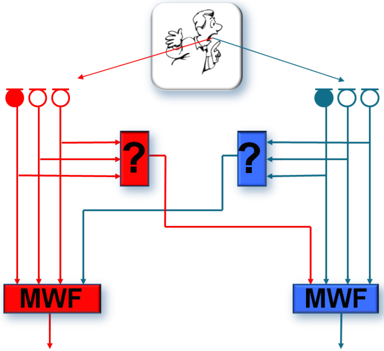
- Two hearing aids (HAs) with wireless link (=2-node WASN)
- Goal: compute MWF including extra signal(s) from other HA
- Each HA uses a local microphone as reference to preserve binaural cues of target speaker



Preliminary case study: binaural hearing aids

Problem statement [Doclo et al., 2009, Srinivasan and Den Brinker, 2009]

- Wireless link only allows exchange of 1 signal (in duplex)
- Which signal should be transmitted?

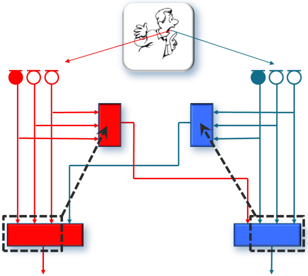


Preliminary case study: binaural hearing aids

Result from [Doclo et al., 2009]

- Copy part of the local MWF coefficients and use it as fusion rule to generate transmit signal (=optimal for single target speaker)
- Iterative computation (details omitted, see later)
- Will extend this result to more general WASN scenarios in this tutorial

PS: similar result exists for binaural MVDR BF [Markovich-Golan et al., 2010]

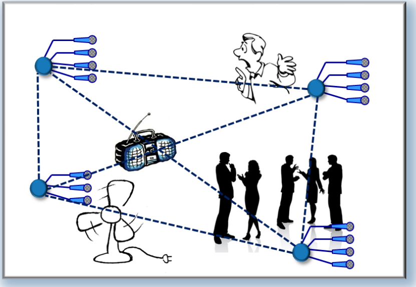


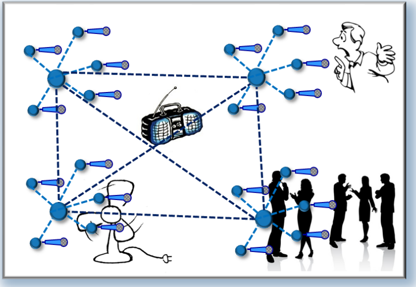
DANSE in fully-connected WASNs

Assumptions:

- Multiple mics per node (array or hierarchical architecture)
- Network is fully connected (=easiest case, will be extended to multi-hop topologies later)
- Each node is a data sink, and requires a node-specific estimate of the target source(s) to preserve spatial cues

  $\Rightarrow$ Distributed adaptive node-specific signal estimation (DANSE)





Notation

- WASN with N nodes $\{1, \ldots, N\} = \mathcal{J}$
- Node $k \in \mathcal{J}$ collects an M_k-channel microphone signal $\mathbf{y}_k(\omega, t)$ (represented in short-time Fourier transform (STFT) domain)
- Will often omit (ω, t) in the sequel for conciseness, keep in mind that all operations are performed in STFT domain.
- Additive noise:

$$\mathbf{y}_k = \mathbf{d}_k + \mathbf{n}_k$$

  $\mathbf{n}_k$ is noise and $\mathbf{d}_k$ is the desired speech signal.
- Stacked vector $\mathbf{y} = [\mathbf{y}_1^T \ \ldots \ \mathbf{y}_N^T]^T$ defines M-channel signal with $M = \sum_{k \in \mathcal{J}} M_k$.
- Similar for $\mathbf{d}$ and $\mathbf{n}$, i.e., $\mathbf{y} = \mathbf{d} + \mathbf{n}$.
- y_{km} denotes the m-th microphone of node k, and $\mathbf{e}_{km} = [0 \ \ldots \ 0 \ 1 \ 0 \ \ldots \ 0]^T$ is a selection vector such that $y_{km} = \mathbf{e}_{km}^T \mathbf{y}$.

Centralized per-node MWFs

- At each node: choose 1st mic as reference microphone (w.l.o.g.)
- Assume all nodes have access to all signals: node $k \in \mathcal{J}$ computes

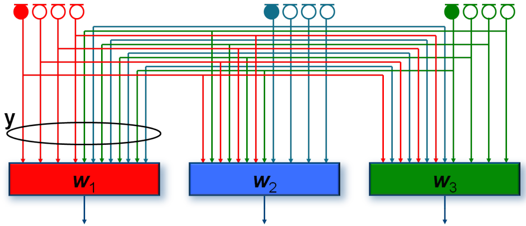
$$\hat{d}_{k1} = \hat{\mathbf{w}}_k^H \mathbf{y}$$

with H denoting conjugate transpose and $\hat{\mathbf{w}}_k$ is node k's MWF

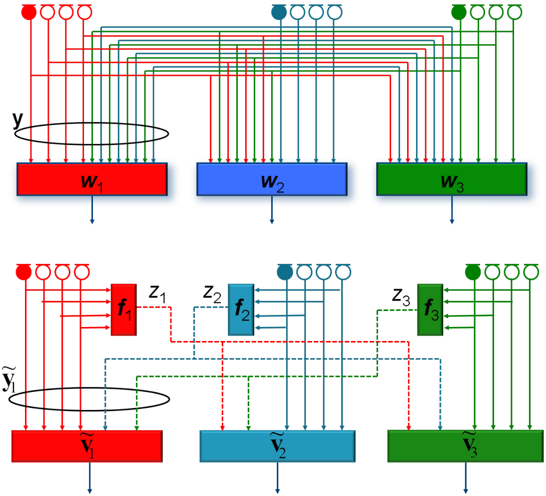
$$\hat{\mathbf{w}}_k = \arg\min_{\mathbf{w}_k} E\{|d_{k1} - \mathbf{w}_k^H \mathbf{y}|^2\} = \mathbf{R}_{yy}^{-1}\mathbf{R}_{dd}\mathbf{e}_{k1}$$

where $\mathbf{R}_{yy} = E\{\mathbf{y}\mathbf{y}^H\}$ and $\mathbf{R}_{dd} = E\{\mathbf{d}\mathbf{d}^H\} = \mathbf{R}_{yy} - \mathbf{R}_{nn}$ (VAD)

PS: will only focus on MWF, but can easily be extended to SDW-MWF.



DANSE signal exchange



DANSE signal exchange

- Node k broadcasts the fused signal z_k to the other nodes:

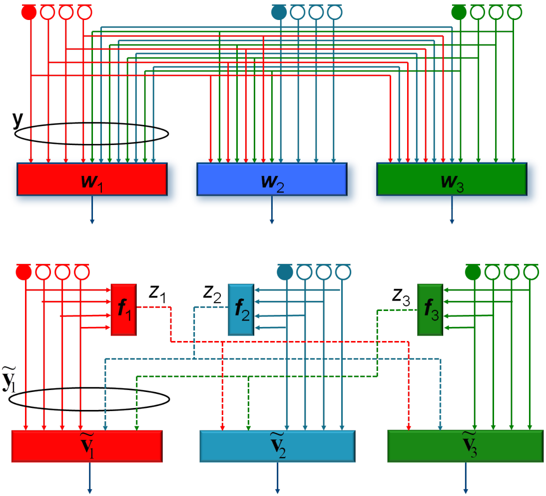
$$z_k^i = \mathbf{f}_k^{i\,H} \mathbf{y}_k$$

  where $\mathbf{f}_k^i$ is an M_k-dimensional fusion vector and i is an iteration index.
- Data compression: M_k-channel signal $\mathbf{y}_k \rightarrow$ single-channel signal z_k^i
- Between iteration i and $i+1$, node k collects samples of

$$\widetilde{\mathbf{y}}_k^i = \begin{bmatrix} \mathbf{y}_k \\ \mathbf{z}_{-k}^i \end{bmatrix} = \widetilde{\mathbf{d}}_k^i + \widetilde{\mathbf{n}}_k^i$$

  with $\mathbf{z}_{-k}^i = [z_1^i \ \ldots \ z_{k-1}^i \ z_{k+1}^i \ \ldots \ z_N^i]^T$.

DANSE signal exchange



DANSE per-node MWFs

- Node k will compute local MWF $\widetilde{\mathbf{v}}_k^i$ that minimizes

$$\min_{\widetilde{\mathbf{v}}_k} E\{|d_{k1} - \widetilde{\mathbf{v}}_k^H \widetilde{\mathbf{y}}_k^i|^2\} \ .$$

- This yields

$$\widetilde{\mathbf{v}}_k^i = \left(\mathbf{R}^i_{\tilde{y}_k\tilde{y}_k}\right)^{-1} \mathbf{R}^i_{\tilde{d}_k\tilde{d}_k} \mathbf{e}_1$$

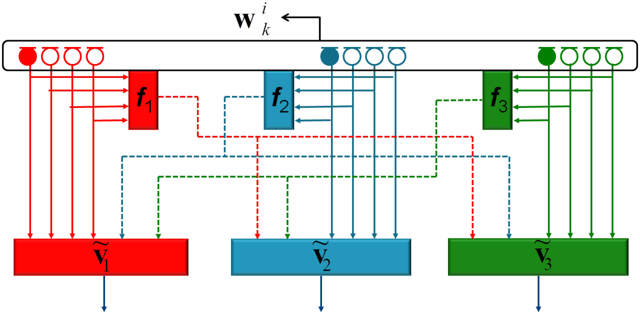
  where $\mathbf{e}_1 = [1\ 0\ \ldots\ 0]$, $\mathbf{R}^i_{\tilde{y}_k\tilde{y}_k} = E\{\widetilde{\mathbf{y}}_k^i \widetilde{\mathbf{y}}_k^{i\,H}\}$, $\mathbf{R}^i_{\tilde{d}_k\tilde{d}_k} = E\{\widetilde{\mathbf{d}}_k^i \widetilde{\mathbf{d}}_k^{i\,H}\}$.
- With VAD: $\mathbf{R}^i_{\tilde{d}_k\tilde{d}_k} = \mathbf{R}^i_{\tilde{y}_k\tilde{y}_k} - \mathbf{R}^i_{\tilde{n}_k\tilde{n}_k}$ (PS: nodes can share VAD info)
- Between iterations i and $i+1$, estimated speech signal at node k:

$$\overline{d}_{k1}^i = \widetilde{\mathbf{v}}_k^{i\,H}\, \widetilde{\mathbf{y}}_k^i$$

Equivalent network-wide filter?

$\Rightarrow$ how does equivalent network-wide filter $\mathbf{w}_k^i$ look like?

$$\overline{d}_{k1}^i = \tilde{\mathbf{v}}_k^{i\,H}\,\tilde{\mathbf{y}}_k^i = \mathbf{w}_k^{i\,H}\,\mathbf{y} \Rightarrow \mathbf{w}_k^i?$$

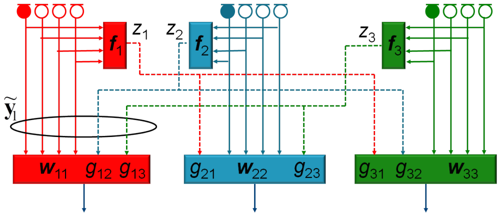


Equivalent network-wide filter?

Local MWF $\leftrightarrow$ network-wide filter

$$\mathbf{w}_1^i = \begin{bmatrix} \mathbf{w}_{11}^i \\ g_{12}^i \mathbf{f}_2^i \\ g_{13}^i \mathbf{f}_3^i \end{bmatrix}, \quad \mathbf{w}_2^i = \begin{bmatrix} g_{21}^i \mathbf{f}_1^i \\ \mathbf{w}_{22}^i \\ g_{23}^i \mathbf{f}_3^i \end{bmatrix}, \quad \mathbf{w}_3^i = \begin{bmatrix} g_{31}^i \mathbf{f}_1^i \\ g_{32}^i \mathbf{f}_2^i \\ \mathbf{w}_{33}^i \end{bmatrix}$$

g_{kq}^i is the coefficient that node k applies to the z_q^i signal from node q.



$$\mathbf{w}_k^i = \begin{bmatrix} \mathbf{w}_{k1}^i \\ \mathbf{w}_{k2}^i \\ \vdots \\ \mathbf{w}_{kk}^i \\ \vdots \\ \mathbf{w}_{kN}^i \end{bmatrix} = \begin{bmatrix} g_{k1}^i \mathbf{f}_1^i \\ g_{k2}^i \mathbf{f}_2^i \\ \vdots \\ \mathbf{w}_{kk}^i \\ \vdots \\ g_{kN}^i \mathbf{f}_N^i \end{bmatrix}$$

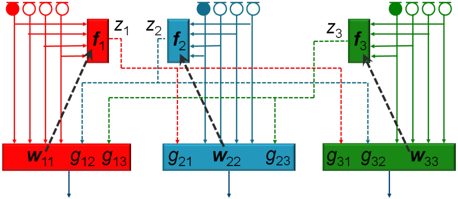
DANSE parametrization

Choice of $\mathbf{f}_k^i$'s

DANSE sets $\mathbf{f}_k^i = \mathbf{w}_{kk}^i$, i.e., $\mathbf{w}_{kk}^i$ serves both as compressor and estimator

$$\mathbf{w}_k^i = \begin{bmatrix} g_{k1}^i \mathbf{w}_{11}^i \\ \vdots \\ g_{kN}^i \mathbf{w}_{NN}^i \end{bmatrix} \quad (g_{kk}^i = 1, \text{ by definition})$$

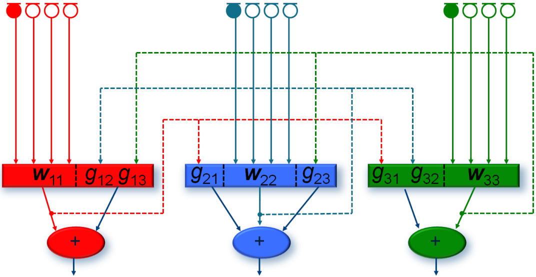
PS: chicken-and-egg problem: need samples of z_k signals to compute local MWFs, but need MWFs to compute samples of z_k's



DANSE parametrization

Example of DANSE parametrization (3-node case)

$$\mathbf{w}_1^i = \begin{bmatrix} \mathbf{w}_{11}^i \\ g_{12}^i \mathbf{w}_{22}^i \\ g_{13}^i \mathbf{w}_{33}^i \end{bmatrix}, \quad \mathbf{w}_2^i = \begin{bmatrix} g_{21}^i \mathbf{w}_{11}^i \\ \mathbf{w}_{22}^i \\ g_{23}^i \mathbf{w}_{33}^i \end{bmatrix}, \quad \mathbf{w}_3^i = \begin{bmatrix} g_{31}^i \mathbf{w}_{11}^i \\ g_{32}^i \mathbf{w}_{22}^i \\ \mathbf{w}_{33}^i \end{bmatrix}$$



PS: similar to $\mathbf{z}_{-k}^i$, introduce notation

$$\mathbf{g}_{k,-k}^i = [g_{k1}^i \ \cdots \ g_{k,k-1}^i \ g_{k,k+1}^i \ \cdots \ g_{kN}^i]^T.$$

Algorithm description (for fixed frequency index ω)

DANSE$_1$ algorithm [Bertrand and Moonen, 2010a]

1. Initialize: $i \leftarrow 0$, $u \leftarrow 1$
   Initialize $\mathbf{w}_{kk}^0$ and $\mathbf{g}_{k,-k}^0$ with random vectors, $\forall\, k \in \mathcal{J}$
2. Each node $k \in \mathcal{J}$ performs the following operation cycle:
   - Collect B new sensor observations $\mathbf{y}_k(\omega, iB+n)$, $n = 0 \ldots B-1$.
   - Compress these M_k-dimensional observations to

$$z_k^i(\omega, iB+n) = \mathbf{w}_{kk}^{i\,H}\mathbf{y}_k(\omega, iB+n), \;\; n = 0 \ldots B-1\,.$$

   - Broadcast B samples of z_k^i to other nodes.
   - Collect B samples of $\mathbf{z}_{-k}^i$ from other nodes.
   - Compute new estimator parameters $\mathbf{w}_{kk}^{i+1}$ and $\mathbf{g}_{k,-k}^{i+1}$ (see next slide).
   - Compute B samples of speech estimate (for $n = 0 \ldots B-1$)

$$\overline{d}_{k1}^i(\omega, iB+n) = \mathbf{w}_{kk}^{i+1\,H}\mathbf{y}_k(\omega, iB+n) + \mathbf{g}_{k,-k}^{i+1\,H}\mathbf{z}_{-k}^i(\omega, iB+n)\,.$$

3. Set $i \leftarrow i+1$, $u \leftarrow (u \mod N)+1$, and return to step 2

Algorithm description (continued)

DANSE_1 algorithm: computation of $\mathbf{w}_{kk}^{i+1}$ and $\mathbf{g}_{k,-k}^{i+1}$

- Node u re-estimates $\mathbf{R}^i_{\tilde{y}_u\tilde{y}_u}$ and $\mathbf{R}^i_{\tilde{d}_u\tilde{d}_u}$, based on the collected samples in $\mathbf{z}^i_{-u}(\omega, iB+n)$ and $\mathbf{y}_u(\omega, iB+n)$, $n = 0 \ldots B-1$.
- $\forall\, k \in \mathcal{J}$, update:

$$\begin{bmatrix} \mathbf{w}_{kk}^{i+1} \\ \mathbf{g}_{k,-k}^{i+1} \end{bmatrix} = \begin{cases} \left(\mathbf{R}^i_{\tilde{y}_k\tilde{y}_k}\right)^{-1} \mathbf{R}^i_{\tilde{d}_k\tilde{d}_k}\mathbf{e}_1 & \text{if } k = u \\ \begin{bmatrix} \mathbf{w}_{kk}^{i} \\ \mathbf{g}_{k,-k}^{i} \end{bmatrix} & \text{if } k \neq u \end{cases}$$

Note:

- Sequential round-robin updating
- B should be large (filters are typically frozen for 1-3 sec)
- Several DANSE algorithms in parallel (one for each frequency bin ω)

Convergence and optimality of DANSE?

Convergence

Does DANSE converge to an equilibrium?
$\Rightarrow$ Does $\lim_{i\to\infty} \mathbf{w}_k^i$ exist, $\forall\, k \in \mathcal{J}$?

Optimality

If DANSE converges to an equilibrium setting, does it have the same estimation performance as the centralized MWF?
$\Rightarrow$ Is $\lim_{i\to\infty} \mathbf{w}_k^i = \hat{\mathbf{w}}_k,\ \forall\, k \in \mathcal{J}$?

1st result

First question: are $\hat{\mathbf{w}}_k$, $\forall\, k \in \mathcal{J}$, in the solution space of DANSE?

Theorem

In case of a single desired speech source, and if all nodes in $\mathcal{J}$ can 'hear' this source, then the solution space defined by the parametrization of DANSE contains the optimal (centralized) MWFs $\hat{\mathbf{w}}_k$, $\forall\, k \in \mathcal{J}$.

Proof outline:

- Single desired speech source:

$$\forall\, k \in \mathcal{J} :\ \mathbf{d}_k(\omega, t) = \mathbf{a}_k(\omega) s(\omega, t)$$

  where $s(\omega, t)$ contains desired speech source and steering vector $\mathbf{a}_k(\omega)$ contains M_k transfer functions from source to M_k microphones.
- Let $\mathbf{a} = [\mathbf{a}_1^T\ \ldots\ \mathbf{a}_N^T]^T$, then $\mathbf{d}(\omega, t) = \mathbf{a}(\omega) s(\omega, t)$.

Proof (continued)

- Centralized MWF at node k:

$$\begin{aligned}\hat{\mathbf{w}}_k &= \mathbf{R}_{yy}^{-1}\mathbf{R}_{dd}\mathbf{e}_{k1} \\ &= \mathbf{R}_{yy}^{-1}\mathbf{a}E\{|s|^2\}\mathbf{a}^H\mathbf{e}_{k1} \\ &= \mathbf{R}_{yy}^{-1}\mathbf{a} \cdot a_{k1}^* E\{|s|^2\}\end{aligned}$$

- It follows that $\forall\, k, q \in \mathcal{J}$:

$$\hat{\mathbf{w}}_k = \alpha_{kq}\hat{\mathbf{w}}_q$$

  with $\alpha_{kq} = \frac{a_{k1}^*}{a_{q1}^*}$.

- In DANSE: set $g_{kq}^i = \alpha_{kq}$ and $\mathbf{w}_{kk}^i = \hat{\mathbf{w}}_{kk}$, $\forall\, k, q \in \mathcal{J}$

$$\forall\, k \in \mathcal{J} : \ \mathbf{w}_k^i = \begin{bmatrix} g_{k1}^i\mathbf{w}_{11}^i \\ \vdots \\ g_{kN}^i\mathbf{w}_{NN}^i \end{bmatrix} = \begin{bmatrix} \alpha_{k1}\hat{\mathbf{w}}_{11} \\ \vdots \\ \alpha_{kN}\hat{\mathbf{w}}_{NN} \end{bmatrix} = \hat{\mathbf{w}}_k \ .$$

2nd result

Theorem (Convergence and optimality of DANSE [Bertrand and Moonen, 2010a])

In case of a single desired speech source, and if $a_{k1} \neq 0$, $\forall\, k \in \mathcal{J}$, then $\lim_{i\to\infty} \mathbf{w}_k^i = \hat{\mathbf{w}}_k$, $\forall\, k \in \mathcal{J}$.

In other words: each node obtains the speech estimate of its corresponding centralized MWF, as if it had access to all the microphone signals.
(proof omitted)

DANSE vs. Centralized MWF

Advantages of DANSE

- Reduced communication bandwidth and reduced transmission energy
- All nodes contribute/cooperate in the processing
  $\Rightarrow$ Small *per-node* processing power
- Inherent dimensionality reduction
  $\Rightarrow$ Many small problems vs. single large problem
  $\Rightarrow$ Often smaller *overall* processing power (due to $O(M^2)$ or $O(M^3)$ complexity)

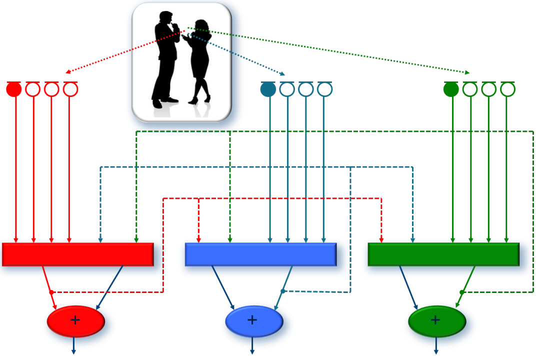
Disadvantages of DANSE

- Reduced tracking performance due to iterative nature (per-node tracking can be improved [Szurley et al., 2013])
- Ripple of errors to other nodes (will be addressed later)

Multiple target speakers

What if desired signal d_{k1} is a mixture of Q desired speech sources?

$\Rightarrow$ $\hat{\mathbf{w}}_k = \alpha_{kq}\hat{\mathbf{w}}_q$ does not hold anymore (see next slide)
$\Rightarrow$ $\hat{\mathbf{w}}_k$ not in solution space of DANSE ☹



Multiple target speakers

- Centralized MWF at node k (for $Q = 2$):

$$\begin{aligned}\hat{\mathbf{w}}_k &= \mathbf{R}_{yy}^{-1}\mathbf{R}_{dd}\mathbf{e}_{k1} \\ &= \mathbf{R}_{yy}^{-1}\,[\mathbf{a}_1\ \mathbf{a}_2]\begin{bmatrix} E\{|s_1|^2\} & 0 \\ 0 & E\{|s_2|^2\}\end{bmatrix}\begin{bmatrix}\mathbf{a}_1^H \\ \mathbf{a}_2^H\end{bmatrix}\mathbf{e}_{k1} \\ &= \mathbf{R}_{yy}^{-1}\,[\mathbf{a}_1\ \mathbf{a}_2]\cdot\mathbf{b}_k\end{aligned}$$

- It follows that $\forall\, k \in \mathcal{J}$:

$$\hat{\mathbf{w}}_k = \mathbf{W}\cdot\mathbf{b}_k$$

with $\mathbf{W} = \mathbf{R}_{yy}^{-1}\,[\mathbf{a}_1\ \ldots\ \mathbf{a}_Q]$ an unknown $M \times Q$ matrix.

Conclusion

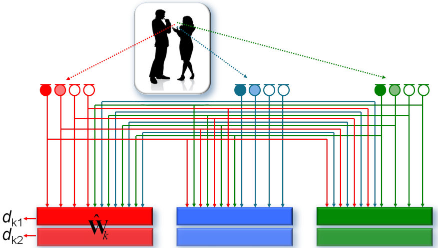
All MWF's $\hat{\mathbf{w}}_k, \forall\, k \in \mathcal{J}$, span a Q-dimensional subspace!
$\Rightarrow$ Need to capture this subspace with DANSE

Generalization: DANSE$_Q$

- Choose $Q-1$ auxiliary reference microphones at each node
- Q-channel desired signal, e.g., $\mathbf{d}_{k,\text{ref}} = [d_{k1} \ \ldots \ d_{kQ}]^T$ (w.l.o.g.)
- Compute Q different MWF's ($M \times Q$ matrix):

$$\hat{\mathbf{W}}_k = \mathbf{R}_{yy}^{-1}\mathbf{R}_{dd}\left[\mathbf{e}_{k1} \ \ldots \ \mathbf{e}_{kQ}\right]$$

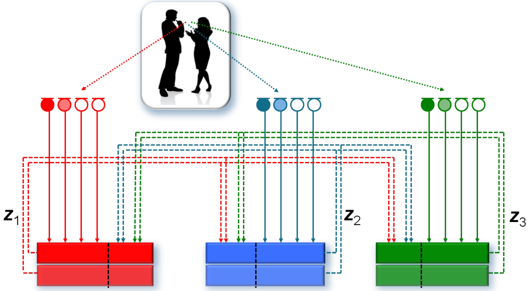
- From previous slide: $\forall\, k, q \in \mathcal{J},\ \exists\, \mathbf{A}_{kq} \in \mathbb{C}^{Q\times Q}:\ \hat{\mathbf{W}}_k = \hat{\mathbf{W}}_q \mathbf{A}_{kq}$.
- If $\mathbf{d}_{k,\text{ref}} = \mathbf{A}_{k,\text{ref}} \cdot \mathbf{s}$, with $\mathbf{A}_{k,\text{ref}} \in \mathbb{C}^{Q\times Q}$ containing the Q-speakers to Q ref.-mic acoustic transfer functions, then $\mathbf{A}_{kq} = \mathbf{A}_{q,\text{ref}}^{-H} \cdot \mathbf{A}_{k,\text{ref}}^{H}$



Generalization: DANSE$_Q$

Q-channel signal broadcasts

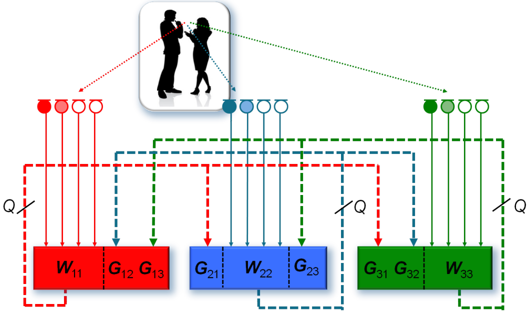
Replace single-channel $z_k^i = \mathbf{w}_{kk}^{i\,H}\mathbf{y}_k$ with a Q-channel signal $\mathbf{z}_k^i = \mathbf{W}_{kk}^{i\,H}\mathbf{y}_k$.
$\Rightarrow$ Communication cost increases linearly with # target speakers



DANSE$_Q$ parametrization

Example of DANSE$_Q$ parametrization (3-node case)

$$\mathbf{W}_1^i = \begin{bmatrix} \mathbf{W}_{11}^i \\ \mathbf{W}_{22}^i\mathbf{G}_{12}^i \\ \mathbf{W}_{33}^i\mathbf{G}_{13}^i \end{bmatrix}, \quad \mathbf{W}_2^i = \begin{bmatrix} \mathbf{W}_{11}^i\mathbf{G}_{21}^i \\ \mathbf{W}_{22}^i \\ \mathbf{W}_{33}^i\mathbf{G}_{23}^i \end{bmatrix}, \quad \mathbf{W}_3^i = \begin{bmatrix} \mathbf{W}_{11}^i\mathbf{G}_{31}^i \\ \mathbf{W}_{22}^i\mathbf{G}_{32}^i \\ \mathbf{W}_{33}^i \end{bmatrix}$$



DANSE$_Q$ parametrization

DANSE$_Q$ parametrization

$$\mathbf{W}_k^i = \begin{bmatrix} \mathbf{W}_{11}^i \mathbf{G}_{k1}^i \\ \vdots \\ \mathbf{W}_{NN}^i \mathbf{G}_{kN}^i \end{bmatrix} \quad \text{where } \mathbf{G}_{kq}^i \in \mathbb{C}^{Q \times Q},\ \mathbf{G}_{kk}^i = \mathbf{I}_Q$$

Since $\forall\, k, q \in \mathcal{J},\ \exists\, \mathbf{A}_{kq} \in \mathbb{C}^{Q \times Q} :\ \hat{\mathbf{W}}_k = \hat{\mathbf{W}}_q \mathbf{A}_{kq}$, the optimal MWF's are in the DANSE solution space (set $\mathbf{W}_{kk}^i = \hat{\mathbf{W}}_{kk}$ and $\mathbf{G}_{kq}^i = \mathbf{A}_{kq}$).

Algorithm description

DANSE$_Q$ algorithm: computation of $\mathbf{W}_{kk}^{i+1}$ and $\mathbf{G}_{k,-k}^{i+1}$

Let $\mathbf{G}_{k,-k}^i = [\mathbf{G}_{k1}^{i\,T} \ \ldots \ \mathbf{G}_{k,k-1}^{i\,T} \ \mathbf{G}_{k,k+1}^{i\,T} \ \ldots \ \mathbf{G}_{kN}^{i\,T}]^T$. Update at node k:

$$\begin{bmatrix} \mathbf{W}_{kk}^{i+1} \\ \mathbf{G}_{k,-k}^{i+1} \end{bmatrix} = \begin{cases} \left(\mathbf{R}_{\tilde{y}_k\tilde{y}_k}^i\right)^{-1} \mathbf{R}_{\tilde{d}_k\tilde{d}_k}^i [\mathbf{e}_1 \ \ldots \ \mathbf{e}_Q] & \text{if } k = u \\ \begin{bmatrix} \mathbf{W}_{kk}^i \\ \mathbf{G}_{k,-k}^i \end{bmatrix} & \text{if } k \neq u \end{cases}$$

where $\widetilde{\mathbf{y}}_k^i$ and $\widetilde{\mathbf{d}}_k^i$ are defined as earlier (but with Q-channel $\mathbf{z}_k^i$ signals).

Convergence and optimality of DANSE$_Q$

Theorem (Convergence and optimality of DANSE$_Q$)

In case of Q desired speech sources, and if $\mathbf{A}_{k,ref}$ is full rank, $\forall\, k \in \mathcal{J}$, then $\lim_{i\to\infty} \mathbf{W}_k^i = \hat{\mathbf{W}}_k$, $\forall\, k \in \mathcal{J}$.

(proof omitted)

Other scenarios

What if the centralized solution is not in DANSE$_Q$ solution space, e.g.,

- DANSE$_Q$ with $Q <$ number of desired speakers?
- DANSE$_Q$ where nodes have 'different interests'

Theorem (Existence of equilibrium [Bertrand and Moonen, 2012b])

Under some technical conditions (details omitted), the DANSE$_Q$ algorithm always has an equilibrium point, i.e., a choice of the local parameters $\mathbf{W}_{kk}^i$ and $\mathbf{G}_{kq}^i$, $\forall\, k, q \in \mathcal{J}$, such that none of the nodes wants to change them.

- Convergence to equilibrium is not proven, but is generally observed in simulations.
- Equilibrium = suboptimal due to selfish updates.
- Game-theoretic framework (selfish nodes) $\rightarrow$ Nash equilibria

Simultaneous node-updating

- In DANSE, the nodes update in a sequential round-robin fashion
  $\Rightarrow$ Slow overall convergence, and slow per-node adaptation
- Can we also let all nodes update simultaneously?
- Sometimes convergence...
- ... but often no convergence ☹ (limit cycle behavior)
- Reason: 'optimal' local update immediately becomes suboptimal due to simultaneous changes in the filters at other nodes
- Solution: Relaxation (details omitted, see [Bertrand and Moonen, 2010b])

$$\mathbf{W}_{kk}^{i+1} = (1-\alpha)\mathbf{W}_{kk}^{i} + \alpha \mathbf{W}_{kk}^{\textit{unrelaxed update}}$$

with $0 < \alpha \leq 1$.

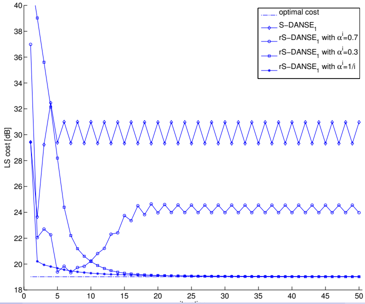
Relaxed simultaneous DANSE (rS-DANSE)

rS-DANSE$_Q$ algorithm: computation of $\mathbf{W}_{kk}^{i+1}$ and $\mathbf{G}_{k,-k}^{i+1}$

Update at all nodes $k \in \mathcal{J}$ simultaneously:

$$\begin{bmatrix} \mathbf{W}_{kk}^{\text{new}} \\ \mathbf{G}_{k,-k}^{i+1} \end{bmatrix} = \left(\mathbf{R}_{\tilde{y}_k\tilde{y}_k}^{i}\right)^{-1} \mathbf{R}_{\tilde{d}_k\tilde{d}_k}^{i} [\mathbf{e}_1 \ \ldots \ \mathbf{e}_Q]$$

$$\mathbf{W}_{kk}^{i+1} = (1-\alpha)\mathbf{W}_{kk}^{i} + \alpha \mathbf{W}_{kk}^{new}$$



Robustified DANSE (R-DANSE)

- Sometimes ill-conditioned nodes:

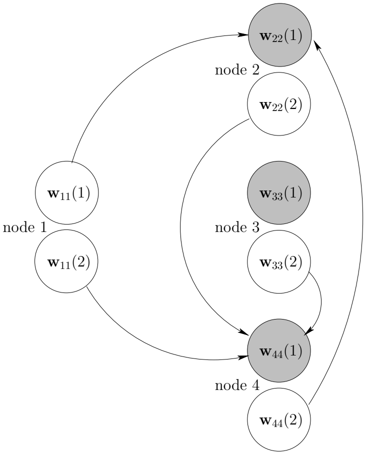
$$a_{k1} \approx 0 \text{ or } \mathbf{A}_{k,\text{ref}} \approx \text{rank deficient}$$

- E.g.: low-SNR node k can be useful as noise reference, but $a_{k1} \approx 0$.
- DANSE suffers from error ripple: erroneous update at one node has an impact on the performance at all other nodes.

Solution

- At ill-conditioned node k: choose z_q^i as reference signal, where node q is a high-SNR node.
- Note: 'desired' signal at node k changes with iteration index i!

Convergence and optimality of R-DANSE



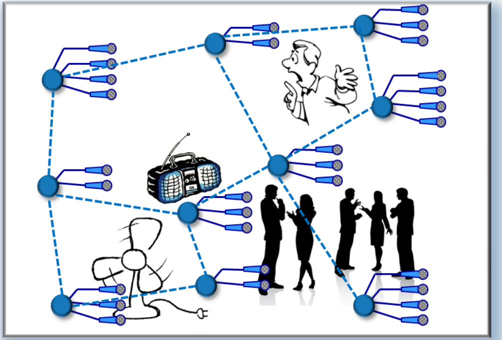
Dependency graph:

- Each column $\mathbf{w}_{kk}^i(m)$ of $\mathbf{W}_{kk}^i$, $\forall\, k \in \mathcal{J}$, $\forall\, m \in \{1, \ldots, Q\}$ is a vertex.
- Note: each $\mathbf{w}_{kk}^i(m)$ corresponds to a particular reference mic
- Draw edge $\mathbf{w}_{kk}^i(m) \rightarrow \mathbf{w}_{qq}^i(n)$ if update of $\mathbf{w}_{kk}^i(m)$ is based on the reference signal $z_q^i(n)$ instead of a local microphone.

If dependency graph contains no loops: convergence and optimality of R-DANSE [Bertrand and Moonen, 2009].

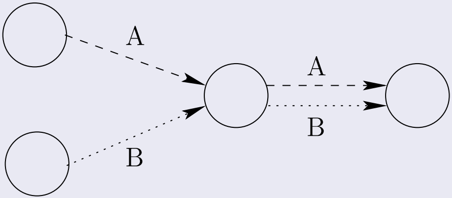
Multi-hop WASNs

- Fully-connected WASNs may require significant transmit power
- Low-power nodes may not be able to reach all other nodes



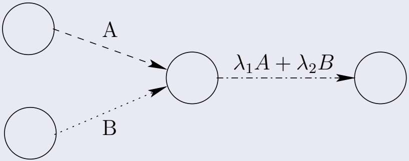
Passing on information

The relay case



- Make network virtually fully connected
- Complex routing problem
- Per-node communication cost grows with network size

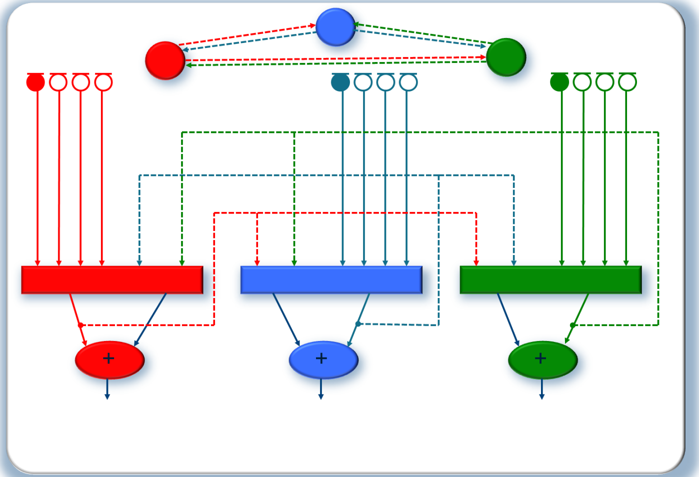
Filter-and-sum combination of inputs



- No routing problems
- Per-node communication cost independent of network size

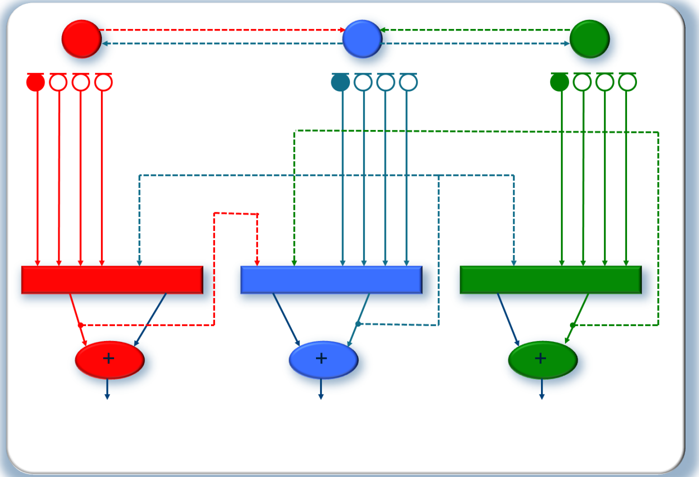
First attempt

Fully-connected DANSE:



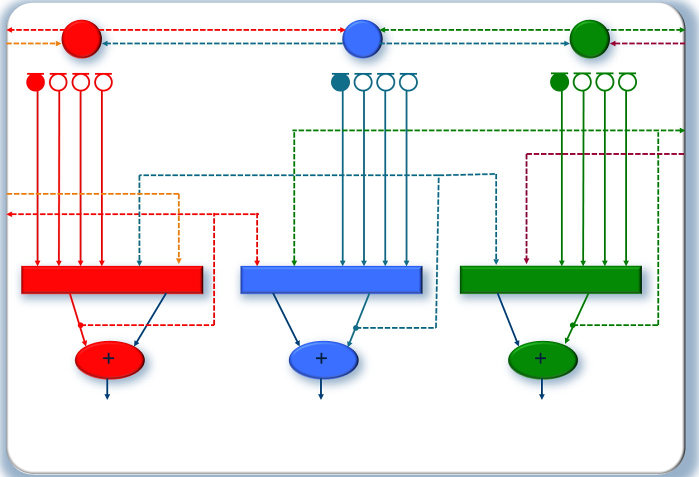
First attempt

Disconnect red and green node...



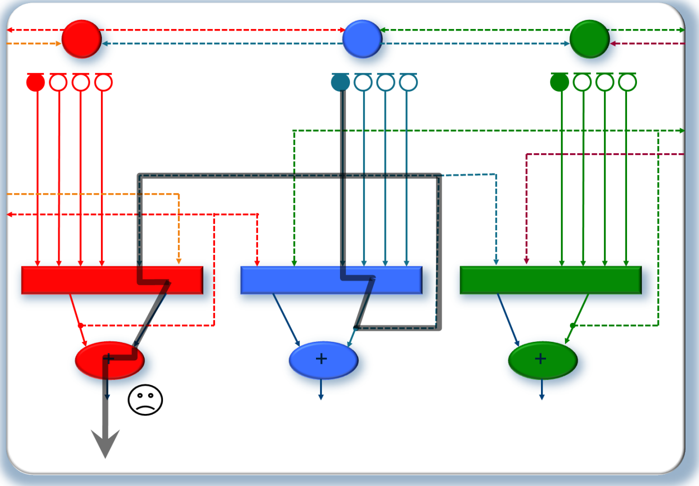
First attempt

... and add new neighbors instead:



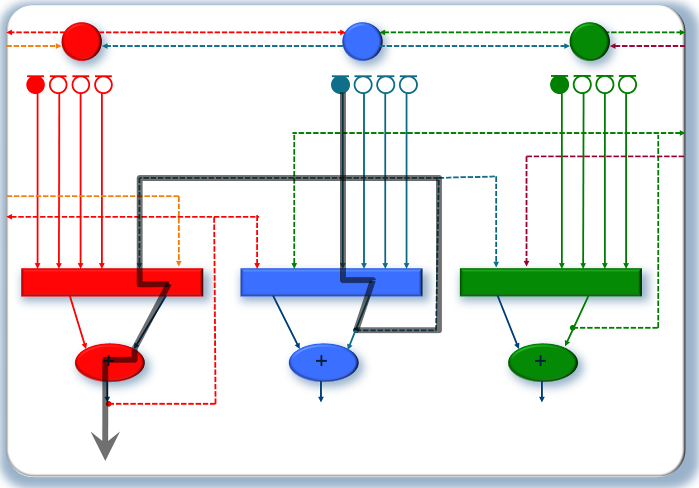
First attempt

Blue node's data is blocked and does not travel beyond red node:



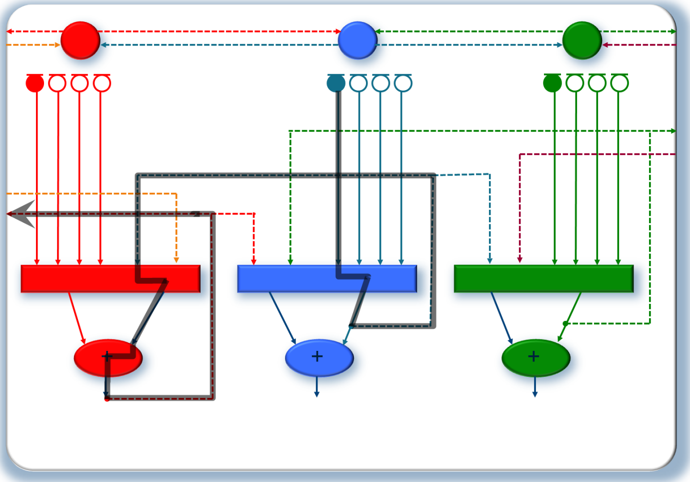
First attempt

Change definition of transmitted signal $\mathbf{z}_k^i$ ('wild guess'):



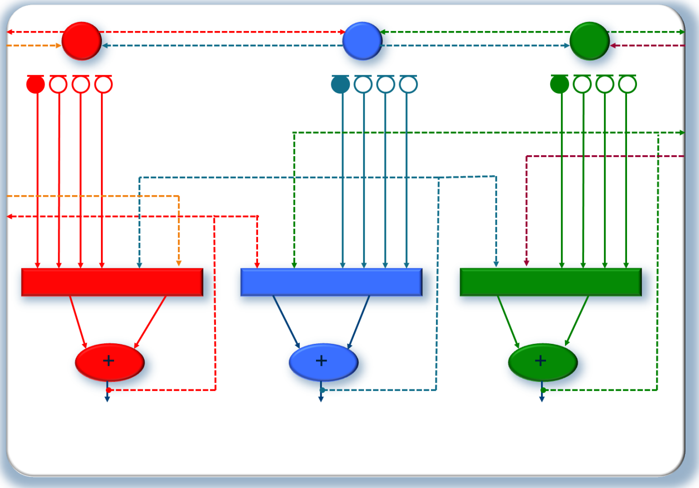
First attempt

Data from blue node travels beyond single-hop region:

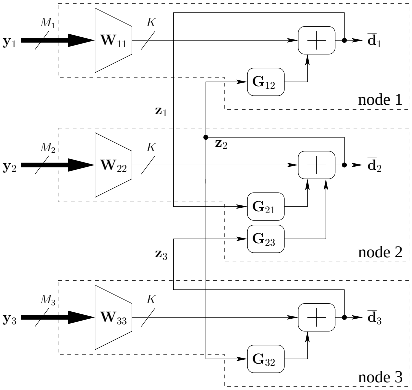


First attempt

Apply similar idea in all nodes:



First attempt



Will this 'wild guess' work???

- $\mathcal{N}_k$= neighbours of k (k excl.)
- Implicit definition of $\mathbf{z}_k^i$:
  $\mathbf{z}_k^i = \mathbf{W}_{kk}^{i\,H}\mathbf{y}_k + \sum_{q\in\mathcal{N}_k} \mathbf{G}_{kq}^{i\,H}\mathbf{z}_q^i$

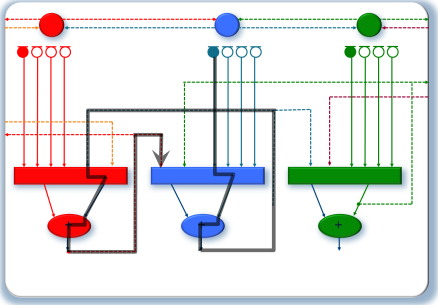
Problem 1: acausality in data flow

Deadlock: nodes wait for each other's **z**-signals

First attempt

Problem 2: feedback

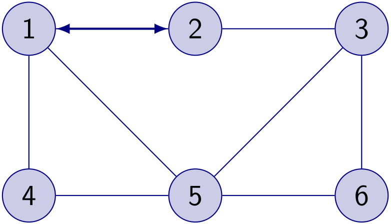
- Feedback path considerably changes algorithm dynamics
- Centralized MWF's are not in solution space (provable)



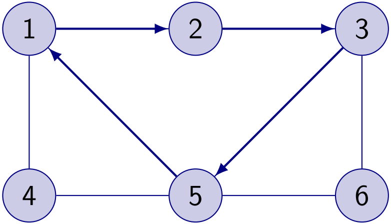
How to get rid of this feedback and causality problem?

2 types of feedback

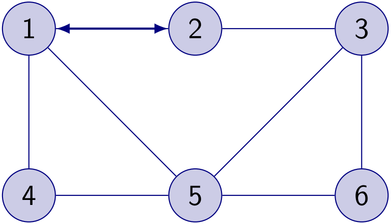
Direct feedback



Indirect feedback



Eliminating direct feedback



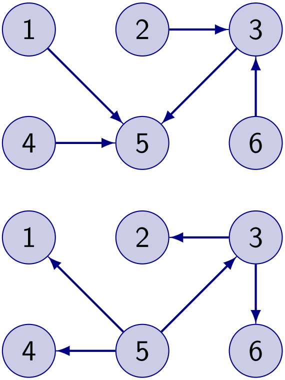
Direct feedback:

- Transmitter feedback cancellation (TFC): send different signal to each neighbour

$$\mathbf{z}_{kq}^{i} = \mathbf{W}_{kk}^{i\,H}\mathbf{y}_k + \sum_{l \in \mathcal{N}_k \backslash \{q\}} \mathbf{G}_{kl}^{i\,H}\mathbf{z}_{lk}^{i}$$

- Better alternative: Receiver feedback cancellation (RFC), i.e., single broadcast signal to all neighbors (details omitted [Bertrand and Moonen, 2011])
- RFC vs. TFC: no influence on algorithm!
  (will assume TFC in sequel w.l.o.g.)

Eliminating indirect feedback

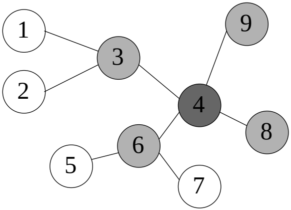


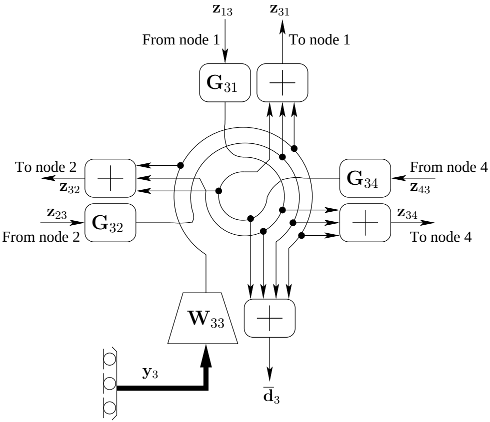
Indirect feedback:

- Prune to tree topology
- In combination with TFC: all feedback eliminated
- Definition of $\mathbf{z}_{kq}^{i}$'s can be resolved:
  - Start at leaf nodes ($|\mathcal{N}_k| = 1$)
  - Leaf node k: $\mathbf{z}_{kq}^{i} = \mathbf{W}_{kk}^{i\,H}\mathbf{y}_k$, i.e., no dependency on other **z**-signals
  - Rest follows in natural order as dictated by the tree
- Similarly, also causality problem in data flow (deadlock) is resolved:
  1. Fusion flow from leaf nodes to root...
  2. ... followed by diffusion flow from root to leafs

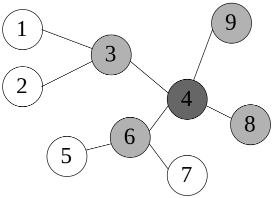
Data-driven signal exchange

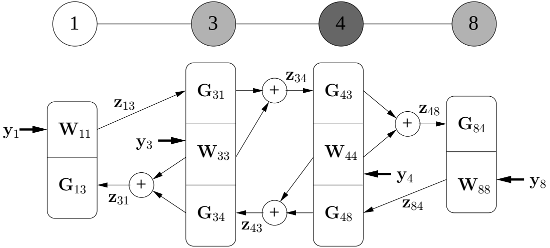
- Data-driven paradigm: each block 'fires' if all of its inputs are available ⇒ no global coordination needed to organize data flow
- Fusion and diffusion flow emerge automatically





Parametrization: example





$$\mathbf{W}_1^i = \begin{bmatrix} \mathbf{W}_{11}^i \\ * \\ \mathbf{W}_{33}^i \mathbf{G}_{13}^i \\ \mathbf{W}_{44}^i \mathbf{G}_{34}^i \mathbf{G}_{13}^i \\ * \\ * \\ * \\ \mathbf{W}_{88}^i \mathbf{G}_{48}^i \mathbf{G}_{34}^i \mathbf{G}_{13}^i \end{bmatrix}$$

$$\mathbf{W}_4^i = \begin{bmatrix} \mathbf{W}_{11}^i \mathbf{G}_{31}^i \mathbf{G}_{43}^i \\ * \\ \mathbf{W}_{33}^i \mathbf{G}_{43}^i \\ \mathbf{W}_{44}^i \\ * \\ * \\ * \\ \mathbf{W}_{88}^i \mathbf{G}_{48}^i \end{bmatrix}$$

General parametrization of Tree-DANSE (T-DANSE)

General parametrization of T-DANSE

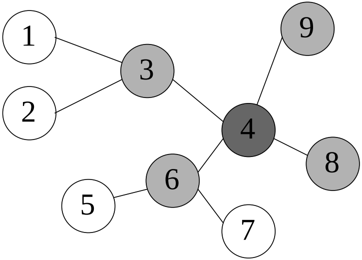
$$\mathbf{W}_k^i = \begin{bmatrix} \mathbf{W}_{11}^i \mathbf{G}_{k \leftarrow 1}^i \\ \vdots \\ \mathbf{W}_{NN}^i \mathbf{G}_{k \leftarrow N}^i \end{bmatrix}$$

- $\mathbf{G}_{p_1 \leftarrow p_t}^i = \mathbf{G}_{p_{t-1} p_t}^i \mathbf{G}_{p_{t-2} p_{t-1}}^i \ldots \mathbf{G}_{p_2 p_3}^i \mathbf{G}_{p_1 p_2}^i$ with order defined by unique path $P_{p_t \rightarrow p_1} = (p_t, p_{t-1}, \ldots, p_2, p_1)$ from p_t to p_1.
- By definition: $\mathbf{G}_{k \leftarrow k}^i = \mathbf{I}_Q$

Compare with fully-connected DANSE:

$$\mathbf{W}_k^i = \begin{bmatrix} \mathbf{W}_{11}^i \mathbf{G}_{k1}^i \\ \vdots \\ \mathbf{W}_{NN}^i \mathbf{G}_{kN}^i \end{bmatrix}$$

Parametrization: example



Complete parametrization of network-wide filter $\mathbf{W}_4^i$:

$$\mathbf{W}_4^i = \begin{bmatrix} \mathbf{W}_{11}^i \mathbf{G}_{4\leftarrow 1}^i \\ \mathbf{W}_{22}^i \mathbf{G}_{4\leftarrow 2}^i \\ \mathbf{W}_{33}^i \mathbf{G}_{4\leftarrow 3}^i \\ \mathbf{W}_{44}^i \\ \mathbf{W}_{55}^i \mathbf{G}_{4\leftarrow 5}^i \\ \mathbf{W}_{66}^i \mathbf{G}_{4\leftarrow 6}^i \\ \mathbf{W}_{77}^i \mathbf{G}_{4\leftarrow 7}^i \\ \mathbf{W}_{88}^i \mathbf{G}_{4\leftarrow 8}^i \end{bmatrix} = \begin{bmatrix} \mathbf{W}_{11}^i \mathbf{G}_{31}^i \mathbf{G}_{43}^i \\ \mathbf{W}_{22}^i \mathbf{G}_{31}^i \mathbf{G}_{43}^i \\ \mathbf{W}_{33}^i \mathbf{G}_{43}^i \\ \mathbf{W}_{44}^i \\ \mathbf{W}_{55}^i \mathbf{G}_{65}^i \mathbf{G}_{46}^i \\ \mathbf{W}_{66}^i \mathbf{G}_{46}^i \\ \mathbf{W}_{77}^i \mathbf{G}_{67}^i \mathbf{G}_{46}^i \\ \mathbf{W}_{88}^i \mathbf{G}_{48}^i \end{bmatrix}$$

Centralized MWF in T-DANSE solution space?

Theorem

In case of Q desired speech sources, and if $\mathbf{A}_{k,ref}$ is full rank, $\forall\, k \in \mathcal{J}$, then the solution space defined by the parametrization of T-DANSE contains the optimal MWFs $\hat{\mathbf{W}}_k$, $\forall\, k \in \mathcal{J}$.

Proof:

- Reminder: $\forall\, k, q \in \mathcal{J}$: $\hat{\mathbf{W}}_k = \hat{\mathbf{W}}_q \mathbf{A}_{kq}$, where

$$\mathbf{A}_{kq} = \mathbf{A}_{q,\text{ref}}^{-H} \cdot \mathbf{A}_{k,\text{ref}}^{H}$$

- Therefore: $\forall\, k, q, n \in \mathcal{J}$: $\mathbf{A}_{nq}\mathbf{A}_{kn} = \mathbf{A}_{kq}$
- Set $\mathbf{G}_{mn}^{i} = \mathbf{A}_{mn}$, then

$$\begin{aligned}\mathbf{G}_{k\leftarrow q}^{i} &= \mathbf{A}_{p_{t-1}q} \cdot \mathbf{A}_{p_{t-2}p_{t-1}} \ldots \mathbf{A}_{p_2 p_3} \cdot \mathbf{A}_{k p_2} \\ &= \mathbf{A}_{kq}\end{aligned}$$

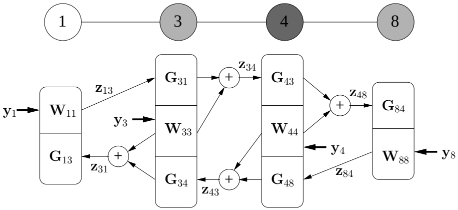
  where $P_{k\leftarrow q} = (q, p_{t-1}, p_{t-2}, \ldots, p_3, p_2, k)$
- Hence, set $\mathbf{W}_{kk}^{i} = \hat{\mathbf{W}}_{kk}$ and $\mathbf{G}_{mn}^{i} = \mathbf{A}_{mn}$, then $\mathbf{W}_{k}^{i} = \hat{\mathbf{W}}_{k}$, Q.E.D.

T-DANSE updating procedure

- Let $\mathbf{z}^i_{\rightarrow k} = [\mathbf{z}^{i\,T}_{n_1} \; \ldots \; \mathbf{z}^{i\,T}_{n_{N_k}}]^T$.
- Node k sets internal fusion rules

$$\mathbf{W}^i_{kk} \text{ and } \mathbf{G}^i_{k,-k} = \left[\mathbf{G}^{i\,T}_{n_1} \; \ldots \; \mathbf{G}^{i\,T}_{n_{N_k}}\right]^T$$

with $n_j \in \mathcal{N}_k$ and $N_k = |\mathcal{N}_k|$.



T-DANSE updating procedure

T-DANSE$_Q$ algorithm: computation of $\mathbf{W}_{kk}^{i+1}$ and $\mathbf{G}_{k,-k}^{i+1}$

- If $k \neq u$, then $\mathbf{W}_{kk}^{i+1} = \mathbf{W}_{kk}^{i}$ and $\mathbf{G}_{k,-k}^{i+1} = \mathbf{G}_{k,-k}^{i}$
- If $k = u$:

$$\begin{bmatrix} \mathbf{W}_{kk}^{i+1} \\ \mathbf{G}_{k,-k}^{i+1} \end{bmatrix} = \underset{\mathbf{W}_{kk}, \mathbf{G}_{k,-k}}{\arg\min} \, E\left\{ \left\| \mathbf{d}_k - \left[\mathbf{W}_{kk}^H \;\; \mathbf{G}_{k,-k}^H \right] \begin{bmatrix} \mathbf{y}_k \\ \mathbf{z}_{\rightarrow k}^i \end{bmatrix} \right\|^2 \right\}$$

$$= \left(\mathbf{R}_{\tilde{y}_k \tilde{y}_k}^i \right)^{-1} \mathbf{R}_{\tilde{d}_k \tilde{d}_k}^i [\mathbf{e}_1 \; \dots \; \mathbf{e}_Q]$$

where $\widetilde{\mathbf{y}}_k^i = [\mathbf{y}_k^T \; \mathbf{z}_{\rightarrow k}^{i\,T}]^T$, and similarly for $\widetilde{\mathbf{d}}_k^i$.

- Identical to fully-connected DANSE updates (but less input signals per node)
- Note: sequential updates (only one node updates in each iteration)

Convergence and optimality of T-DANSE

Theorem (Convergence and optimality of T-DANSE [Bertrand and Moonen, 2011]**)**

In case of Q desired speech sources, if $\mathbf{A}_{k,ref}$ is full rank, $\forall\, k \in \mathcal{J}$, and if the node-per-node updating order of T-DANSE is defined by a path through the network that visits all nodes, then $\lim_{i\to\infty} \mathbf{W}_k^i = \hat{\mathbf{W}}_k$, $\forall\, k \in \mathcal{J}$.

- Note: updating order must follow a path through the network
- Random order updating also works in general, but no proof
- However: path-based updating converges faster (experimental observation)

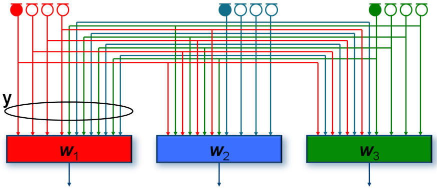
LCMV beamforming revisited

Centralized node-specific LCMV BF at node k:

$$\hat{\mathbf{w}}_k = \arg\min_{\mathbf{w}_k} \left(\mathbf{w}_k^H \mathbf{R}_{yy} \mathbf{w}_k, \text{ s.t. } \mathbf{A}^H \mathbf{w}_k = \mathbf{f}_k \right)$$

$$= \mathbf{R}_{yy}^{-1} \mathbf{A} \left(\mathbf{A}^H \mathbf{R}_{yy}^{-1} \mathbf{A} \right)^{-1} \mathbf{f}_k$$

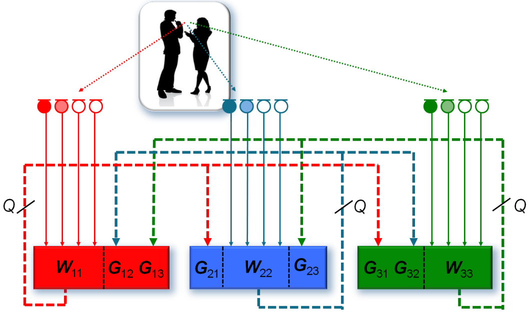
- $\mathbf{A}$ $M \times Q$ steering matrix from Q 'relevant' sources to M microphones
- $\mathbf{f}_k$ node-specific response for each of the Q sources
- Relevant sources may also contain interferers!



PS: Will assume in sequel that **A** is known. For unknown **A**, refer to [Markovich et al., 2009] or [Bertrand and Moonen, 2012a]

Linearly-constrained DANSE (LC-DANSE)

- DANSE ↔ (SDW-)MWF
- LC-DANSE ↔ LCMV
- Similar idea, similar block scheme



Note: Q is # constraints

Linearly-constrained DANSE (LC-DANSE)

- $\hat{\mathbf{w}}_k = \mathbf{R}_{yy}^{-1}\mathbf{A}\left(\mathbf{A}^H\mathbf{R}_{yy}^{-1}\mathbf{A}\right)^{-1}\mathbf{f}_k$
  $\Rightarrow$ joint Q-dim subspace: $\hat{\mathbf{w}}_k = \mathbf{W}\cdot\mathbf{f}_k,\ \forall\, k \in \mathcal{J}$.
- Add $Q-1$ auxiliary LCMV-problems:

$$\hat{\mathbf{W}}_k = \underset{\mathbf{W}_k}{\arg\min}\ \left(\mathrm{Tr}\left(\mathbf{W}_k^H\mathbf{R}_{yy}\mathbf{W}_k\right),\ \text{s.t. } \mathbf{A}^H\mathbf{W}_k = \mathbf{F}_k\right)$$
$$= \mathbf{R}_{yy}^{-1}\mathbf{A}\left(\mathbf{A}^H\mathbf{R}_{yy}^{-1}\mathbf{A}\right)^{-1}\mathbf{F}_k$$

  with $\mathbf{F}_k$ a $Q\times Q$ matrix of full rank, with $\mathbf{f}_k$ in first column.
- $\forall\, k, q \in \mathcal{J}$: $\hat{\mathbf{W}}_k = \hat{\mathbf{W}}_q\mathbf{A}_{kq}$ with

$$\mathbf{A}_{kq} = \mathbf{F}_q^{-1}\mathbf{F}_k$$

Conclusion: Centralized LCMV solutions are in (LC-)DANSE solution space! (set $\mathbf{W}_{kk}^i = \hat{\mathbf{W}}_{kk}$ and $\mathbf{G}_{kq}^i = \mathbf{A}_{kq}$)

Linearly-constrained DANSE (LC-DANSE)

Match constraints with compressed signals:

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_N \end{bmatrix} \leftrightarrow \mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \vdots \\ \mathbf{A}_N \end{bmatrix}$$

$$\widetilde{\mathbf{y}}_k^i = \begin{bmatrix} \mathbf{y}_k \\ \mathbf{z}_{-k}^i \end{bmatrix} \leftrightarrow \widetilde{\mathbf{A}}_k^i = \begin{bmatrix} \mathbf{A}_k \\ \mathbf{C}_{-k}^i \end{bmatrix}$$

$$\mathbf{z}_{-k}^i = \begin{bmatrix} \mathbf{z}_1^i \\ \vdots \\ \mathbf{z}_{k-1}^i \\ \mathbf{z}_{k+1}^i \\ \vdots \\ \mathbf{z}_N^i \end{bmatrix} \leftrightarrow \mathbf{C}_{-k}^i = \begin{bmatrix} \mathbf{C}_1^i \\ \vdots \\ \mathbf{C}_{k-1}^i \\ \mathbf{C}_{k+1}^i \\ \vdots \\ \mathbf{C}_N^i \end{bmatrix}$$

$$\mathbf{z}_k^i = \mathbf{W}_{kk}^{i\,H} \mathbf{y}_k \leftrightarrow \mathbf{C}_k^i = \mathbf{W}_{kk}^{i\,H} \mathbf{A}_k$$

LC-DANSE Algorithm description

LC-DANSE$_Q$ algorithm: computation of $\mathbf{W}_{kk}^{i+1}$ and $\mathbf{G}_{k,-k}^{i+1}$

Update at node k:

$$\begin{bmatrix} \mathbf{W}_{kk}^{i+1} \\ \mathbf{G}_{k,-k}^{i+1} \end{bmatrix} = \begin{cases} \left(\mathbf{R}_{\tilde{y}_k\tilde{y}_k}^i\right)^{-1} \widetilde{\mathbf{A}}_k^i \left(\widetilde{\mathbf{A}}_k^{i\,H} \left(\mathbf{R}_{\tilde{y}_k\tilde{y}_k}^i\right)^{-1} \widetilde{\mathbf{A}}_k^i\right)^{-1} \mathbf{F}_k & \text{if } k = u \\ \begin{bmatrix} \mathbf{W}_{kk}^i \\ \mathbf{G}_{k,-k}^i \end{bmatrix} & \text{if } k \neq u \end{cases}$$

Note: computation of $\widetilde{\mathbf{A}}_k^i$ requires exchange of $\mathbf{W}_{kk}^i$'s. However, filter coefficients are typically frozen for some time (2-3s), hence negligible compared to data rate of $\mathbf{z}_k^i$'s.

LC-DANSE: final remarks

- Provable convergence and optimality
- Further reading: [Bertrand and Moonen, 2012a]
- Q constraints $\Rightarrow$ Q-channel broadcast signals
- If node-specific aspect is removed (same $\mathbf{f}_k$ in all nodes): single-channel z_k^i's are sufficient! [Bertrand and Moonen, 2013]
- Related GSC implementation: [Markovich-Golan et al., 2013] (covered in part III)

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