Multi-Microphone Speaker Localization on Manifolds

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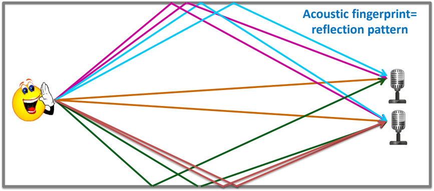
Acoustic Source Localization & Tracking

Goal

Locate/track a sound source(s) given a set of microphone signals in acoustic environment

Environment-aware data-driven acoustic source localization

- Based on fingerprints in acoustic enclosures
- Exploiting the availability of multiple microphones in ad hoc networks of low-end devices
- Utilizing the power of modern data-driven paradigms



The Need

We live in a noisy and reverberant world

1. Train stations
2. Factories
3. Cars/Homes/Offices
4. Cocktail parties

Adverse acoustic conditions

1. Noise
2. Reverberation
3. Competing speakers

Speaker Localization

An Essential component in Speech Processing Applications

1. Hands-free voice communication
2. Human-car communication
3. Camera steering
4. Robot audition
5. Smart homes and smart conference call systems
6. Assistive devices for the elderly ("Aging in Place")
7. Smart speakers, e.g. Amazon Echo, Google Home and Apple HomePod
8. Personal assistant, e.g. Apple Siri, Cortana Microsoft and Google Assistant
9. Hearing aids
10. Hearables (wireless earbuds, augmented hearing)

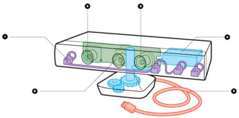
Many Microphones are Available

Devices equipped with multiple microphones

1. Cellular phones
2. Laptops and tablets
3. Hearing devices
4. Smart watches
5. Smart glasses
6. Smart homes & cars







Speaker Localization and Tracking

Basics and Prior Art

- The target of localization (or tracking) algorithms can be either the coordinates of the speaker, or the time difference of arrival (TDOA) between microphone signals
- The mathematical relations between the coordinates of the speakers (or the respective TDOAs) and the observed signals is nonlinear and non-injective
- Localization approaches can be roughly split into two groups:
  - Single-step approaches: The location of the source is estimated directly from the microphone signals
  - Dual-step approaches: TDOAs between pairs of microphone are first estimated, and are subsequently merged to obtain the source coordinates by intersecting geometric surfaces

Speaker Localization and Tracking (cont.)

Basics and Prior Art

- Dynamic scenarios further complicates the problem, as smoothness of the speaker trajectory should be kept
- Multiple concurrent speakers scenarios are even more challenging, due to mixing between the reflections of all speakers (in this tutorial this issue will only be briefly addressed)
- The shortcomings of classical localization and tracking methods may be alleviated by harnessing data-driven methodologies

Speaker Localization and Tracking (cont.)

Basics and Prior Art

Single-step

- MUSIC [Schmidt, 1986]; used as a baseline for LOCATA challenge [Löllmann et al., 2018]
- ESPRIT [Roy and Kailath, 1989]; applied to speech signals (e.g. [Teutsch and Kellermann, 2005]) or as features for subsequent spatial processing (e.g. [Thiergart et al., 2014])
- Steered-response beamformer phase transform (SRP-PHAT) [DiBiase et al., 2001, Do et al., 2007]; can also be used as features for subsequent spatial processing (e.g. [Madhu and Martin, 2018, Hadad and Gannot, 2018])
- Maximum-Likelihood (e.g. [Yao et al., 2002])

Speaker Localization and Tracking (cont.)

Basics and Prior Art

TDOA estimation and tracking

- Generalized cross-correlation (GCC) [Knapp and Carter, 1976]
- Subspace methods [Benesty, 2000, Doclo and Moonen, 2003]
- Relative transfer function (RTF)-based [Dvorkind and Gannot, 2005]

Geometric intersections

- Linear intersections [Brandstein et al., 1997]
- Spherical intersections [Schau and Robinson, 1987]
- Spherical interpolation [Smith and Abel, 1987]
- One-step least squares (OSLS) [Huang et al., 2000]
- Linear-correction least-squares [Huang et al., 2001]

Speaker Localization and Tracking (cont.)

Basics and Prior Art

Bayesian

- Extended, Unscented and Iterated-Extended Kalman filter [Gannot and Dvorkind, 2006, Faubel et al., 2009, Klee et al., 2006]
- Particle filters (PF), Rao-Blackwellised Monte-Carlo [Ward et al., 2003, Lehmann and Williamson, 2006, Zhong and Hopgood, 2008, Levy et al., 2011]
- Variational Bayes [Ban et al., 2019, Soussana and Gannot, 2019]
- Probability hypothesis density (PHD) filters [Evers and Naylor, 2017]
- Viterbi algorithm for Hidden Markov model (HMM) [Roman et al., 2003]

Speaker Localization and Tracking (cont.)

Basics and Prior Art

Non-Bayesian

- Mixture of Gaussians (MoG) clustering of SRP outputs with expectation-maximization (EM) [Madhu et al., 2008]; using binaural cues and MoG clustering with predefined grid positions as Gaussian centroids [Mandel et al., 2007, Mandel et al., 2010]; using mixture of von Mises distribution [Brendel et al., 2018]
- RANdom SAmple Consensus (RANSAC) and EM [Traa and Smaragdis, 2014]
- Recursive [Schwartz and Gannot, 2013] and distributed [Dorfan and Gannot, 2015, Dorfan et al., 2018] EM MoG clustering with predefined grid positions as Gaussian centroids
- EM with spectrogram clustering [Dorfan et al., 2016, Schwartz et al., 2017, Weisberg et al., 2019]

Speaker Localization and Tracking (cont.)

Basics and Prior Art

Learning-based methods

- Probabilistic piecewise affine mapping based on smooth binaural manifolds of low dimensions
  [Deleforge and Horaud, 2012, Deleforge et al., 2013, Deleforge et al., 2015]
- MoG clustering of binaural cues using multi-condition training
  [May et al., 2011]
- Gaussian processes inference to map coherent-to-diffuse power ratio and source distance [Brendel and Kellermann, 2019]
- Deep learning for classifying feature vectors to candidate positions: Fully connected [Xiao et al., 2015]; convolutional neural networks (CNN) [Takeda and Komatani, 2016, Chakrabarty and Habets, 2019], convolutional recurrent neural network (CRNN) [Adavanne et al., 2018, Perotin et al., 2019]
- Deep ranking using triplet loss [Opochinsky et al., 2019]

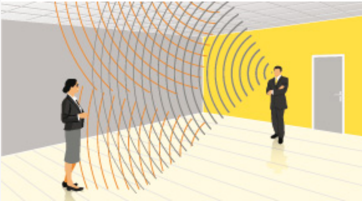
Our Proposed Methodology

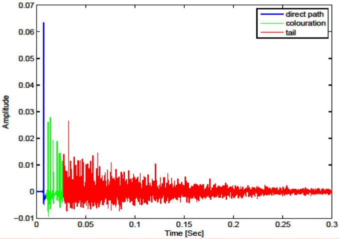
- Utilizes the reflection pattern of the acoustic propagation
- Harnesses the power of machine learning (specifically, manifold learning) to deal with the complexity of the acoustic propagation
- Is suitable for both coordinate localizing and TDOA estimation, depending on the number of nodes used
- Can be also used in dynamic scenarios

Room Acoustics Essentials

Acoustic propagation models

- When sound propagates in an enclosure it undergoes reflections from its surfaces
- Reflections can be modeled as images beyond room walls and hence impinging the microphones from many directions [Allen and Berkley, 1979, Peterson, 1986]
- Statistical models for late reflections [Polack, 1993, Schroeder, 1996, Jot et al., 1997]
- Late reflections tend to be diffused, hence do not exhibit directionality [Dal Degan and Prati, 1988, Habets and Gannot, 2007]





Describing the wave propagation of an audio source in an arbitrary acoustic environment is a cumbersome task

How to Utilize the Intricate Reflection Pattern?

- Classical multi-microphone speech processing algorithms, and specifically acoustic source localization, model the acoustic propagation as time difference of arrival (TDOA)-only, while ignoring sound reflections and focusing only on the-direct path
- It was shown [Gannot et al., 2001, Markovich et al., 2009] that utilizing the entire acoustic propagation path, manifested by the acoustic impulse response (AIR), may significantly improve the performance of speech processing algorithms
- We will show that the intricate acoustic reflection patterns define a fingerprint, uniquely characterizing the source location in the enclosure

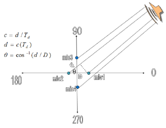
How to Model the Acoustic Environment?

How to model the acoustic path?

Classical speech processing

TDOA-only

Too simplified



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Classical speech processing

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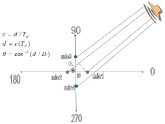
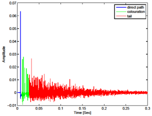


To cope with reverberations

Entire Impulse Response

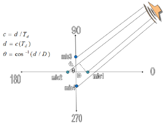
Perfect but Complicated: No simple Model

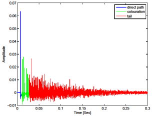


How to Model the Acoustic Environment?

How to model the acoustic path?

Classical speech processing		To cope with reverberations
TDOA-only	Manifold Learning	Entire Impulse Response
Too simplified	Simple to describe and to implement	Perfect but Complicated: No simple Model





How to Harness Manifold Learning to Infer Source Location from Acoustic Relection Pattern?

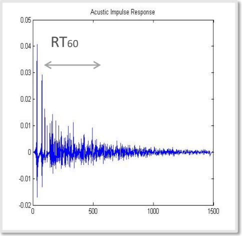
- As shown above, describing the wave propagation of an audio source in an arbitrary acoustic environment is, a cumbersome task, since:
  - No simple mathematical models exist
  - The estimation of the vast number of parameters used to describe the wave propagation suffers from large errors
- We will show that the collection of acoustic fingerprints pertain to a low-dimensional acoustic manifold:
  - The intrinsic degrees of freedom (DoF) in acoustic responses are limited to a small number of variables (e.g. room dimensions, source and microphone positions, and refection coefficients)
  - In a fixed environment and microphone constellation, the acoustic responses intrinsically differ only by the source position

How to Harness Manifold Learning to Infer Source Location from Acoustic Relection Pattern? (cont.)

Manifold learning: A data-driven approach

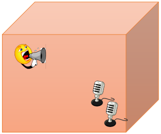
→ Extracts the geometrical structure of the acoustic fingerprints

→ Can reveal the controlling DoFs and hence improve localization ability

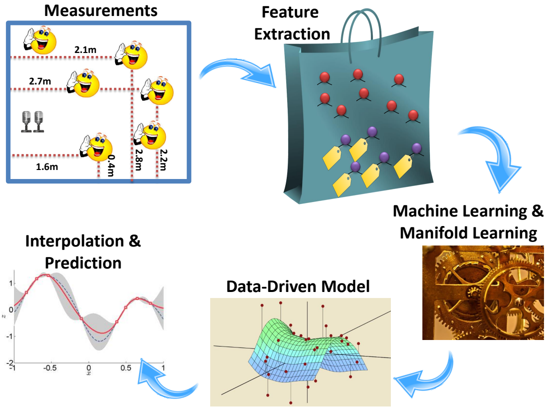


Controlling Parameters

- Room dimensions
- Reverberation time
- Microphone position
- …
- Source position



The Data Processing Pipeline



- Data pre-processing and feature extraction
- Analyzing the geometric structure of the data (manifold learning)
- Deriving data-driven algorithms and inference methodologies to perform a certain task (in our case, localizing the source)

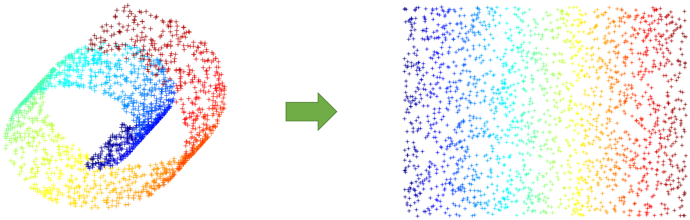
Outline

Outline

Data Representation

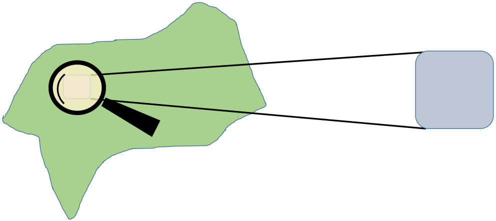
- Measured data often exhibit highly redundant representations
- Often controlled by a small set of parameters
- Lie on a low dimensional manifold

- Consider n high-dimensional features $\mathbf{h}_i \in \mathbb{R}^D$ extracted from the data
- Construct a low-dimensional representation $\mathbf{y}_i \in \mathbb{R}^d$ of $\mathbf{h}_i$, $d < D$, respecting the manifold geometric structure



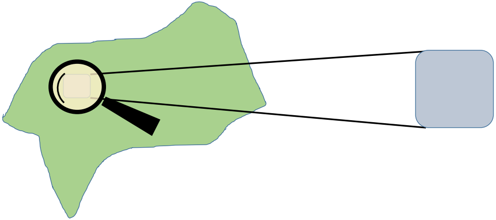
What is a manifold?

- A topological space in which every local region is isomorphic to a Euclidean space
- Differential manifold: a manifold that is locally similar to a linear space
- Riemannian manifold: a differential manifold equipped with an inner product (metric) defined on the tangent plane to the manifold at every point



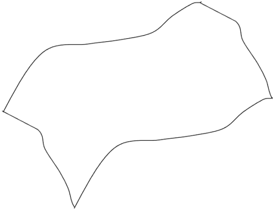
Laplacian

- The Laplacian Δ is an operator defined by the divergence of the gradient of a function in a Euclidean space: $\Delta = \nabla \cdot \nabla$
- The Laplace–Beltrami operator $\mathcal{L}$ is the extension to Riemannian manifolds
- It was shown [Bérard et al., 1994] that a local coordinate system can be built using the Laplacian of the manifold
  $\Rightarrow$ The Laplacian contains all the information about the manifold geometry



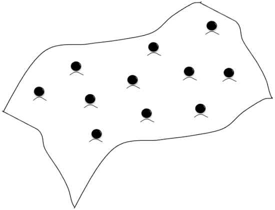
Discretization of the Manifold

- The Laplacian is an infinite-dimension operator defined on continuous spaces



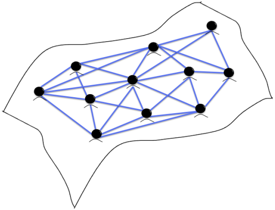
Discretization of the Manifold

- The Laplacian is an infinite-dimension operator defined on continuous spaces
  - We are typically given a finite set of observations in discrete spaces
  - What is the finite-dimension counterpart of the Laplacian?



Discretization of the Manifold

- The Laplacian is an infinite-dimension operator defined on continuous spaces
  - We are typically given a finite set of observations in discrete spaces
  - What is the finite-dimension counterpart of the Laplacian?
- The manifold can be empirically represented by a graph
  - The observations are the graph nodes
  - Define a finite operator (matrix) – the graph Laplacian



Manifold Learning Paradigms

Why learning?

- Given high-dimensional point clouds
- Recall: assume they lie on a manifold, but no other prior knowledge
- The goal is to recover the manifold from the data

Classical methods

- The foundations of manifold learning were laid in 2000:
  - Locally linear embedding (LLE) [Roweis and Saul, 2000]
  - Isometric feature mapping (ISOMAP) [Tenenbaum et al., 2000]
- We will focus on diffusion maps, [Coifman and Lafon, 2006] due to the notion of diffusion distance

Locally-Linear Embedding [Roweis and Saul, 2000]

- Determine the neighbours $\mathcal{N}_i$ of each point $\mathbf{h}_i$
- Compute the weights that best reconstruct each point from its neighbors by minimizing:

$$E(\mathbf{W}) = \sum_i \|\mathbf{h}_i - \sum_{j \in \mathcal{N}_i} W_{ij}\mathbf{h}_j\|^2$$

such that $\sum_{j \in \mathcal{N}_i} W_{ij} = 1$

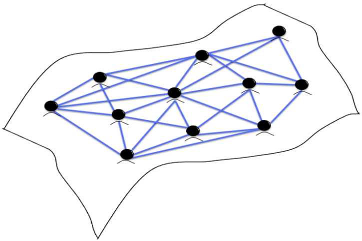
- Compute a low-dimensional embedding $\mathbf{y}_i \in \mathbb{R}^d$ of $\mathbf{h}_i \in \mathbb{R}^D$ by:

$$\underset{\mathbf{y}_i}{\operatorname{argmin}} \sum_i \|\mathbf{y}_i - \sum_j W_{ij}\mathbf{y}_j\|^2$$

  - $\mathbf{W}$ is an $n \times n$ sparse matrix
  - The embedding can be obtained by solving a sparse eigenvalue problem

ISOMAP [Tenenbaum et al., 2000]

- Determine the neighbours $\mathcal{N}_i$ of each point $\mathbf{h}_i$
- Construct a neighborhood graph:
  - Each point $\mathbf{h}_i$ is a graph node (vertex)
  - Node $\mathbf{h}_i$ is connected by an edge to each neighbor $\mathbf{h}_j \in \mathcal{N}_i$

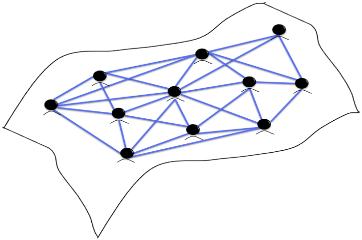


ISOMAP [Tenenbaum et al., 2000]

- Compute the shortest path between any two nodes d_{ij} (number of edges)
- Compute a low-dimensional embedding with multidimensional scaling (MDS) [Kruskal, 1964] by:

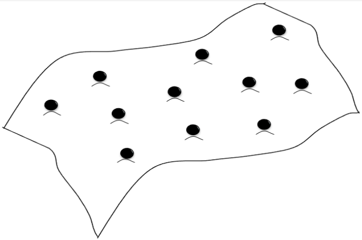
$$\text{argmin}_{\mathbf{y}_1,\ldots,\mathbf{y}_n \in \mathbb{R}^d} \sum_{i<j} (\|\mathbf{y}_i - \mathbf{y}_j\| - d_{ij})^2$$

- Can be solved by eigenvalue decomposition (EVD) of a matrix computed from the pairwise distances $d_{i,j}$



Diffusion Maps [Coifman and Lafon, 2006]

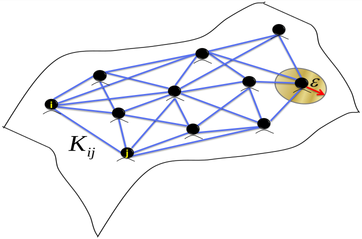
- Samples are the graph nodes



Diffusion Maps [Coifman and Lafon, 2006]

- Samples are the graph nodes
- The weights of the edges are defined using a kernel function:

$$K_{ij} = k(\mathbf{h}_i, \mathbf{h}_j) = \exp\left\{-\frac{\|\mathbf{h}_i - \mathbf{h}_j\|^2}{\varepsilon}\right\}$$



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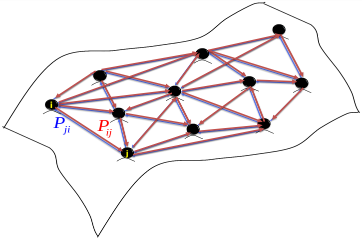
- Define a Markov process on the graph by the transition matrix:

$$P_{ij} = p(\mathbf{h}_i, \mathbf{h}_j) = K_{ij} / \sum_{r=1}^{N} K_{ir}$$

which is a discretization of a diffusion process on the manifold



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- In matrix form: $\mathbf{P} = \mathbf{D}^{-1}\mathbf{K} \in \mathbb{R}^{n \times n}$
where $\mathbf{D}$ is diagonal with:

$$D_{ii} = \sum_{r=1}^{n} K_{ir}$$

- $\mathbf{P}$ is similar to a symmetric matrix $\mathbf{S} = \mathbf{D}^{-1/2}\mathbf{K}\mathbf{D}^{-1/2}$ by

$$\mathbf{P} = \mathbf{D}^{-1/2}\mathbf{S}\mathbf{D}^{1/2}$$

so $\mathbf{P}$ has a real spectrum

Diffusion Maps [Coifman and Lafon, 2006]

- The (normalized) graph Laplacian is defined by

$$\mathbf{N} = \mathbf{I} - \mathbf{P}$$

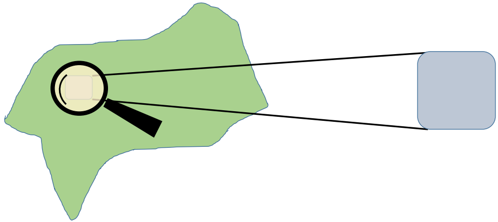
- It was shown that $\mathbf{N}$ asymptotically ($\varepsilon \to 0$ $n \to \infty$) converges to the Laplacian $\mathcal{L}$
  $\Rightarrow$ The normalized graph Laplacian $\mathbf{N}$ (and $\mathbf{P}$) contains the information about the manifold geometry

Diffusion Maps [Coifman and Lafon, 2006]

- Apply eigenvalue decomposition (EVD) to the matrix $\mathbf{P} \in \mathbb{R}^{n \times n}$ and obtain n eigenvalues $\{\lambda_j\}$ and n right eigenvectors $\{\varphi_j\}$ in $\mathbb{R}^n$
- A nonlinear mapping into a new d-dimensional Euclidean space:

$$\mathbf{\Phi}_d : \mathbf{h}_i \mapsto \left[\lambda_1 \varphi_1(i), \ldots, \lambda_d \varphi_d(i)\right]^T$$

where $d < n$ is typically set by prior knowledge or according to a "spectral gap"



Q: In what sense the space is Euclidean?

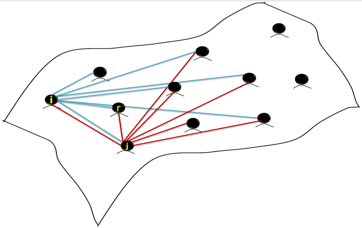
Diffusion Distance

The distance along the manifold is approximated by the diffusion distance:

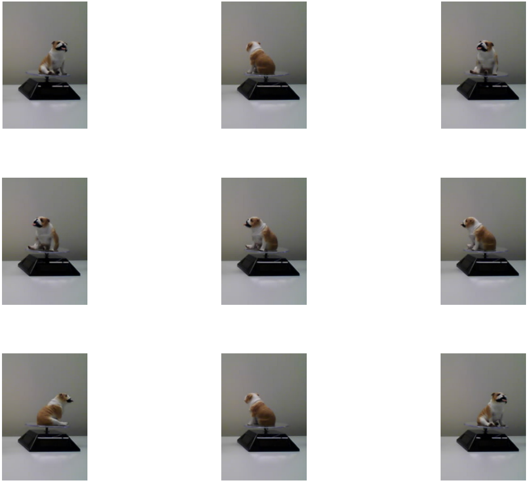
$$D^2_{\mathrm{Diff}}(\mathbf{h}_i, \mathbf{h}_j) = \sum_{r=1}^{N} \left(p\left(\mathbf{h}_i, \mathbf{h}_r\right) - p\left(\mathbf{h}_j, \mathbf{h}_r\right)\right)^2 / \phi_0^{(r)}$$

- Two points are close if they are highly connected in the graph
- The diffusion distance can be well approximated by the Euclidean distance in the embedded domain:

$$D_{\mathrm{Diff}}(\mathbf{h}_i, \mathbf{h}_j) \cong \|\mathbf{\Phi}_d(\mathbf{h}_i) - \mathbf{\Phi}_d(\mathbf{h}_j)\|$$



Toy Example



Building the embedding

Diffusion maps

- Compute $\mathbf{P}$ (or equivalently $\mathbf{N}$) from the images $\mathbf{h}_i$
- Apply EVD to $\mathbf{P}$ (or $\mathbf{N}$) and obtain eigenvalues $\{\lambda_j\}$ and eigenvectors $\{\varphi_j\}$
- Build the map:

$$\mathbf{h}_i \mapsto [\lambda_1\varphi_1(i),\ \lambda_2\varphi_2(i)]$$

where here $\lambda_1 = \lambda_2$

Geometry of Data

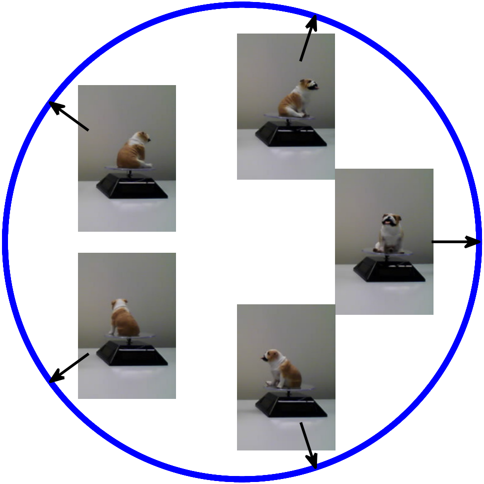


Figure: Each sample (snapshot) is a point on the circle (the rotation angle)

Geometry of Data

Video: One variable.

Geometry of Data

Video: One variable.

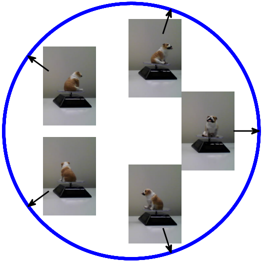
Q: why a circle?

Diffusion

Analogy to the toy example

- The manifold $\mathcal{M}$ is a 1-dimensional sphere in $\mathbb{R}$
- Can be parametrized by $x_i \in [0, 2\pi]$ representing the hidden angle (with periodic boundary conditions)
- We have access to the images $\mathbf{h}_i$, which can be viewed as functions of the hidden angle

$$\mathbf{h}_i := h(x_i)$$



Diffusion

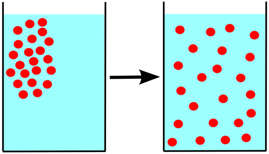
Diffusion process

- The Laplace-Beltrami operator defines a diffusion process on the manifold:

$$u_t = \mathcal{L}u$$

for a function $u(t,x)$ defined on the manifold, $x \in \mathcal{M}$ and $t \geq 0$

- Suppose $u(0,x) = f(x)$
$\Rightarrow u(t,x)$ is the propagation of $f(x)$ by the application of $\mathcal{L}$



Diffusion

The 1D case

$$u_t = \mathcal{L}u = u_{xx}$$
$$u(x, 0) = f(x), \forall x \in [0, 1]$$
$$u(0, t) = u(1, t), u_x(0, t) = u_x(1, t), \forall t > 0$$

Diffusion

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Solution I: separation of variables

$$u(x,t) = X(x)T(t)$$
$$\frac{\dot{T}(t)}{T(t)} = \frac{X''(x)}{X(x)} = -\lambda$$
$$X''(x) = -\lambda X(x)$$

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$$\lambda_k = 4k^2\pi^2;\ k = 1, 2, \ldots$$

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Solution II: EVD

$$\mathcal{L}X(x) = X''(x) = -\lambda X(x)$$

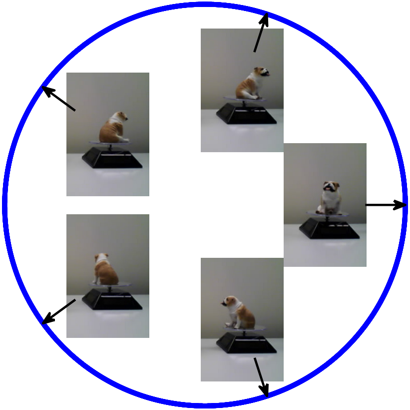
Therefore, λ_k and $X_k(x)$ are the eigenvalues and eigenfunctions of $\mathcal{L}$, respectively

$$X_k(x) = \sin(\sqrt{\lambda_k}x), \cos(\sqrt{\lambda_k}x)$$
$$\lambda_k = 4k^2\pi^2;\ k = 1, 2, \ldots$$

Diffusion

The embedding

$$\mathbf{h}_i \mapsto [4\pi^2 \cos(2\pi x_i), 4\pi^2 \sin(2\pi x_i)]$$



Smoothness on the Manifold

Measuring smoothness over $\mathcal{M}$:

- Let $\mathbf{h} \in \mathcal{M}$ and $f : \mathcal{M} \to \mathbb{R}$
- The gradient $\nabla f(\mathbf{h})$ represents amplitude and direction of variation of f around $\mathbf{h}$
- A global measure of smoothness for f:

$$\|f\|_{\mathcal{M}}^2 = \int_{\mathcal{M}} \|\nabla f(\mathbf{h})\|^2 d\mu(\mathbf{h})$$

where $\mu(\mathbf{h})$ is the probability measure of $\mathbf{h}$ on $\mathcal{M}$

Smoothness on the Manifold

Measuring smoothness on $\mathcal{M}$:

- Stokes' theorem links gradient and Laplacian:

$$\int_{\mathcal{M}} \|\nabla f(\mathbf{h})\|^2 d\mu(\mathbf{h}) = \int_{\mathcal{M}} f(\mathbf{h})\mathcal{L}f(\mathbf{h})d\mu(\mathbf{h}) = \langle f(\mathbf{h}), \mathcal{L}f(\mathbf{h})\rangle$$

where $\mathcal{L} = \nabla \cdot \nabla$ is the Laplace-Beltrami ("Laplacian") operator

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Smoothness on the manifold: Discretization

- The graph Laplacian discretization: $\mathbf{L} \triangleq \mathbf{D} - \mathbf{K}$
- $\mathbf{P} = \mathbf{D}^{-1}\mathbf{K}$ and $\mathbf{N} = \mathbf{D}^{-1}\mathbf{L} = \mathbf{I} - \mathbf{P}$
- Smoothness of $\mathbf{f} = [f(\mathbf{h}_1), ..., f(\mathbf{h}_n)]$ on the graph: $\mathbf{f}^T\mathbf{L}\mathbf{f} = \langle \mathbf{f}, \mathbf{L}\mathbf{f}\rangle$

Smoothness on the Manifold: Discretization

- Further insight can be obtained by:

$$
\begin{aligned}
\mathbf{f}^T\mathbf{L}\mathbf{f} &= \sum_{i,j=1}^{n} f(\mathbf{h}_i)L_{ij}f(\mathbf{h}_j) \\
&= \sum_{i=1}^{n}\left(\sum_{j=1}^{n} K_{ij} - K_{ii}\right) f^2(\mathbf{h}_i) - \sum_{\substack{i,j=1 \\ i\neq j}}^{n} K_{ij}f(\mathbf{h}_i)f(\mathbf{h}_j) \\
&= \sum_{i,j=1}^{n} K_{ij}f^2(\mathbf{h}_i) - \sum_{i,j=1}^{n} K_{ij}f(\mathbf{h}_i)f(\mathbf{h}_j) \\
&= \frac{1}{2}\sum_{i,j=1}^{n} K_{ij}\left(f(\mathbf{h}_i) - f(\mathbf{h}_j)\right)^2
\end{aligned}
$$

- When K_{ij} is large, the mappings $f(\mathbf{h}_i)$ and $f(\mathbf{h}_j)$ are "encouraged" to be close

Further Insight

Eigenvalue decomposition of the Laplacian

- Recall: $\mathbf{L}$ is the symmetric graph Laplacian with eigenvalues $0 = \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$ and corresponding eigenvectors $\varphi_1, \ldots, \varphi_n$
- By the Courant-Fischer Theorem:

$$\lambda_k = \min_{\mathbf{f} \perp \varphi_1, \ldots, \varphi_{k-1}} \frac{\mathbf{f}^T \mathbf{L} \mathbf{f}}{\mathbf{f}^T \mathbf{D} \mathbf{f}}$$

$$\varphi_k = \operatorname*{argmin}_{\mathbf{f} \perp \varphi_1, \ldots, \varphi_{k-1}} \frac{\mathbf{f}^T \mathbf{L} \mathbf{f}}{\mathbf{f}^T \mathbf{D} \mathbf{f}}$$

Analogy to the Fourier transform:

- Small eigenvalues correspond to eigenvectors that change slowly on the manifold ("low frequencies")
- Large eigenvalues correspond to eigenvectors that change rapidly on the manifold ("high frequencies")

Laplacian Eigenmaps [Belkin and Niyogi, 2003]

Building low-dimensional embedding

- Similarly to Diffusion Maps:

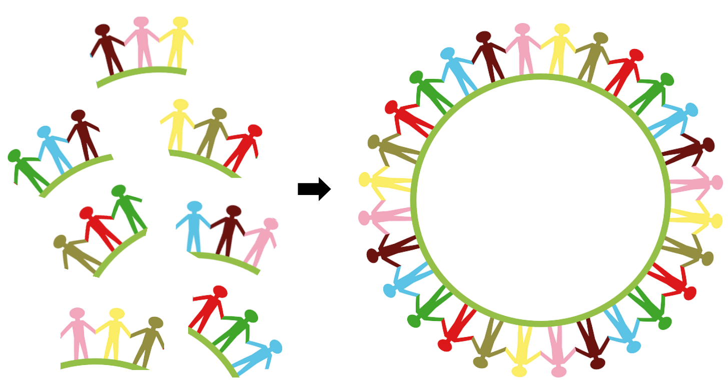
$$\mathbf{h}_i \mapsto [\varphi_1(i), \ldots, \varphi_d(i)]^T$$

- As shown above, the Euclidean distance between embedded points respects the similarity defined by the kernel
  - High kernel affinity leads to nearby embedded points

Manifold Learning – Summary

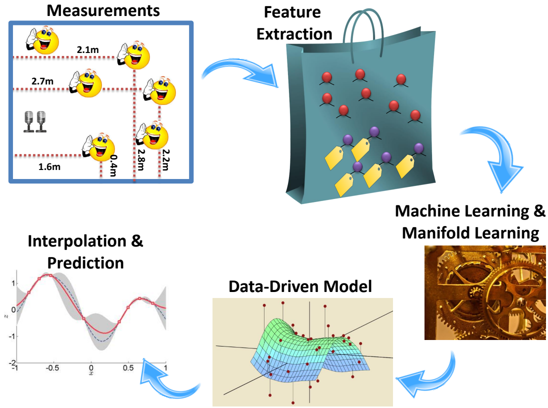
"Tell me who your friends are and I will tell you who you are"

- In high-dimensional space only local relations are meaningful
- Find a global fit that preserves local relations:
  - Local relations by kernel function similarity
  - Global fit by spectral decomposition



Outline

The Data Processing Pipeline



- Data pre-processing and feature extraction
- Analyzing the geometric structure of the data (manifold learning)
- Deriving data-driven algorithms and inference methodologies to perform a certain task (in our case, localizing the source)

Data Model: The Two Microphone Case

Microphone signals:

The measured signals in the two microphones (an extension to multiple microphone pairs will be discussed later):

$$y_1(n) = a_1(n) * s(n) + u_1(n)$$
$$y_2(n) = a_2(n) * s(n) + u_2(n)$$

- $s(n)$ - the source signal
- $a_i(n),\ i = \{1, 2\}$ - the acoustic impulse responses relating the source and each of the microphones
- $u_i(n),\ i = \{1, 2\}$ - noise signals, independent of the source

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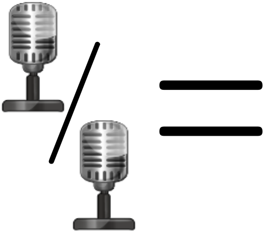
Find a feature vector representing the characteristics of the acoustic path and independent of the source signal

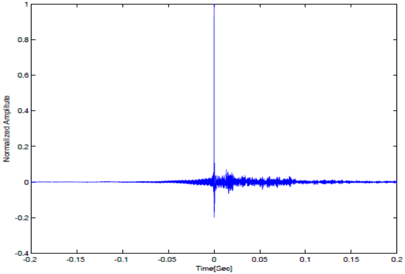
The Features

Alternatives

- The relative transfer function (RTF) for pairs of microphones
  [Gannot et al., 2001]
- Power ratios of directional microphone (using a microphone quartet)
  [Laufer-Goldshtein et al., 2018a]
- Relative harmonic coefficients (using spherical microphone array)
  [Hu et al., 2019]

Relative Transfer Function (RTF) [Gannot et al., 2001]





RTF:

- Defined as the ratio between the transfer functions of the two mics:

$$H_{12}(k) = \frac{A_2(k)}{A_1(k)} \overset{\text{low-noise}}{\simeq} \frac{\hat{S}_{y_2 y_1}(k)}{\hat{S}_{y_1 y_1}(k)}$$

  estimated based on PSD and cross-PSD
- Define the feature vector: $\mathbf{h} = [\hat{H}_{12}(k_1), \ldots, \hat{H}_{12}(k_D)]^T$

Relative Transfer Function (RTF) [Gannot et al., 2001]

$$= f\left(\begin{array}{l}\text{room dimensions}\\ \text{reverberation time}\\ \text{microphone position}\\ \text{source position}\\ \ldots\end{array}\right)$$

RTF:

- Represents the acoustic paths and is independent of the source signal
- Generalizes the TDOA
- Depends on a small set of parameters related to the physical characteristics of the environment

Relative Transfer Function (RTF) [Gannot et al., 2001]

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- ...

)

RTF:

- Represents the acoustic paths and is independent of the source signal
- Generalizes the TDOA
- Depends on a small set of parameters related to the physical characteristics of the environment
- In a static environment the source position is the only varying degree of freedom

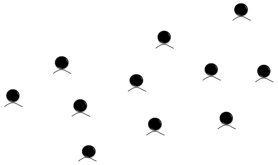
A plethora of methods for RTF Estimation

- Utilizing speech non-stationarity and noise stationarity [Shalvi and Weinstein, 1996]; [Gannot et al., 2001]
- Extension to two nonstationary sources in stationary noise [Reuven et al., 2008]
- Subspace tracking for single speaker [Affes and Grenier, 1997]
- GEVD analysis for multiple speakers [Markovich et al., 2009]
- Subspace tracking for multiple speakers [Markovich-Golan et al., 2010]
- Utilizing RIR Sparseness [Koldovký et al., 2015]
- Utilizing BSS methods [Reindl et al., 2013]
- Applying covariance whitening or covariance subtraction [Markovich-Golan et al., 2018]
- Utilizing speech sparsity in the STFT domain (w-disjoint orthogonality [Yilmaz and Rickard, 2004]) and Simplex analysis [Laufer-Goldshtein et al., 2018c]

Outline

How to Measure the Affinity between Two RTF Samples? [Laufer-Goldshtein et al., 2015]

The RTFs are represented as points in a high dimensional space



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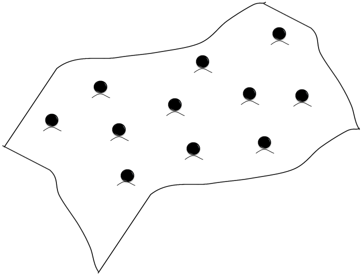

Acoustic manifold

- They lie on a low dimensional nonlinear manifold $\mathcal{M}$

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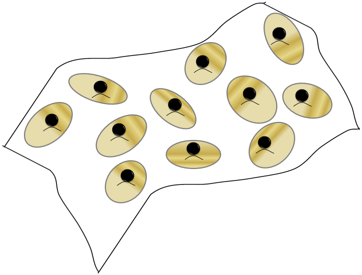

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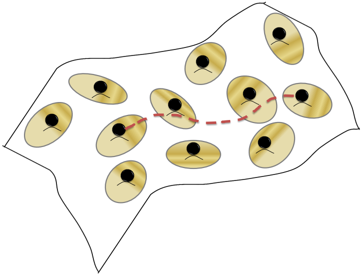
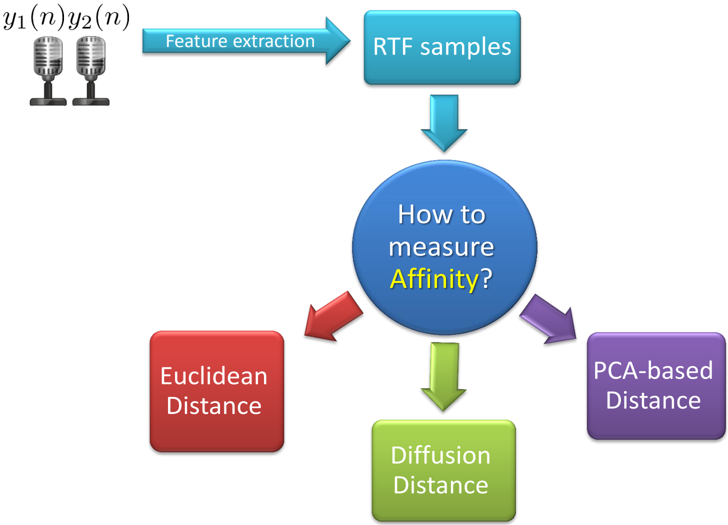


Acoustic manifold

- They lie on a low dimensional nonlinear manifold $\mathcal{M}$
- Linearity is preserved in small neighbourhoods
- Distances between RTFs should be measured along the manifold



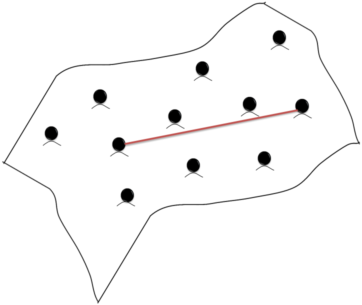
Each distance measure relies on a different hidden assumption about the underlying structure of the RTF samples

Euclidean Distance

The Euclidean distance between RTFs

$$D_{\text{Euc}}(\mathbf{h}_i, \mathbf{h}_j) = \|\mathbf{h}_i - \mathbf{h}_j\|$$

- Compares two RTFs in their original space
- Does not assume an existence of a manifold
- Respects flat manifolds



A good affinity measure only when the RTFs are uniformly scattered all over the space, or when they lie on a flat manifold

Principal component analysis (PCA) [Jolliffe, 2011]

PCA algorithm

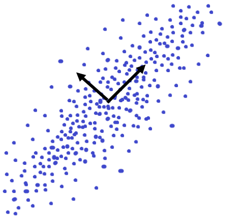
- Find the vector that maximizes the variance of the data:

$$\underset{||\mathbf{y}||^2=1}{\operatorname{argmax}} \mathbf{y}^T \hat{\mathbf{R}} \mathbf{y}$$

  where $\hat{\mathbf{R}}$ is the sample covariance matrix of the data
- The first principal component corresponds to the first eigenvector of $\hat{\mathbf{R}}$
- The kth principal component corresponds to the kth eigenvector of $\hat{\mathbf{R}}$

Linear vs. nonliner

- PCA - smoothness over sample covariance
- Laplacain Eigenmaps - smoothness over graph Laplacain



PCA-Based Distance

PCA algorithm

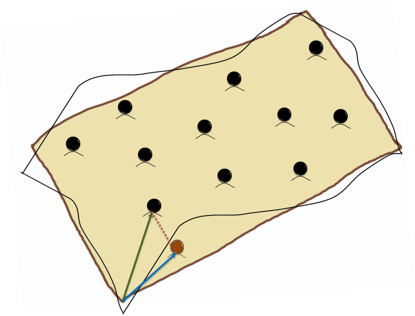
- The principal components - the d dominant eigenvectors $\{\mathbf{v}_i\}_{i=1}^{d}$ of the covariance matrix of the data
- The RTFs are linearly projected onto the principal components:

$$\boldsymbol{\nu}(\mathbf{h}_i) = [\mathbf{v}_1, \ldots \mathbf{v}_d]^T (\mathbf{h}_i - \boldsymbol{\mu})$$

PCA-based distance between RTFs

$$D_{\mathrm{PCA}}(\mathbf{h}_i, \mathbf{h}_j) = \|\boldsymbol{\nu}(\mathbf{h}_i) - \boldsymbol{\nu}(\mathbf{h}_j)\|$$

- A global approach - extracts principal directions of the entire set
- Linear projections - the manifold is assumed to be linear/flat



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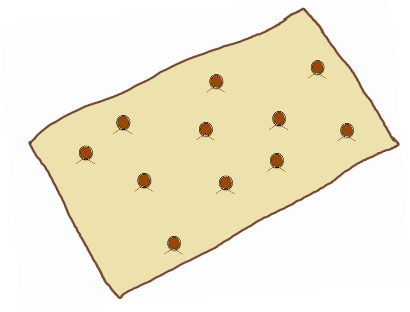

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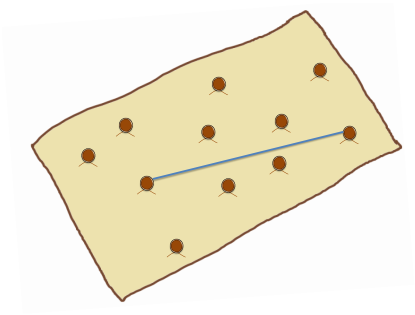
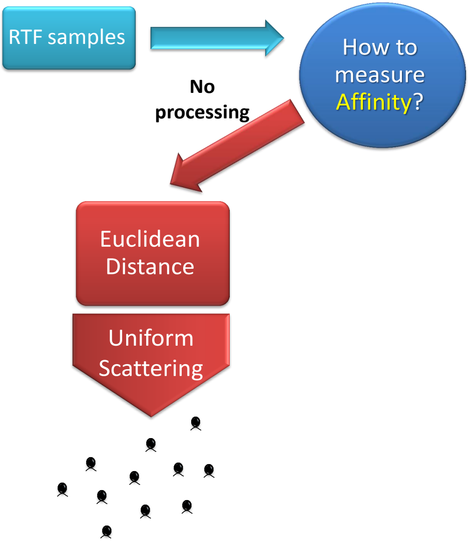
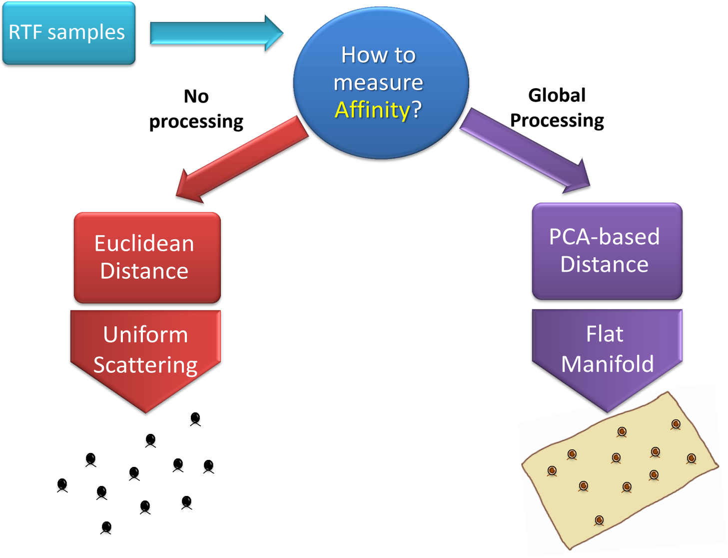



RTF samples
How to measure Affinity?
No processing
Euclidean Distance
Uniform Scattering


RTF samples
How to measure Affinity?
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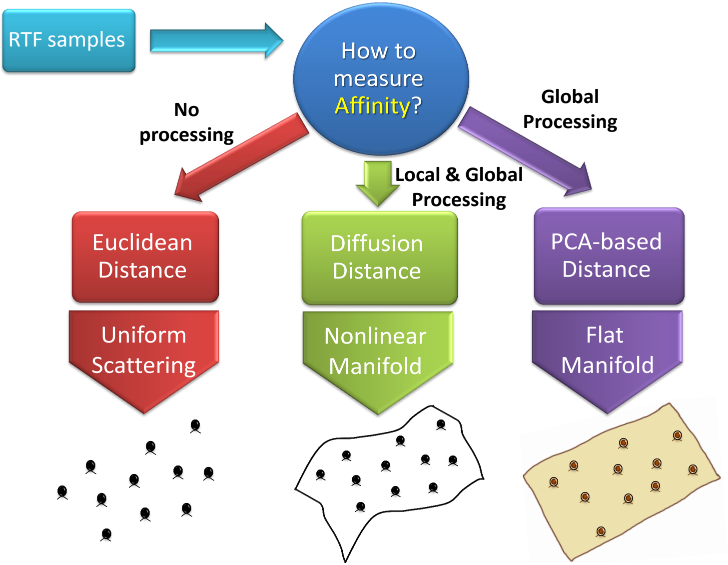
Global Processing
Euclidean Distance
Uniform Scattering
PCA-based Distance
Flat Manifold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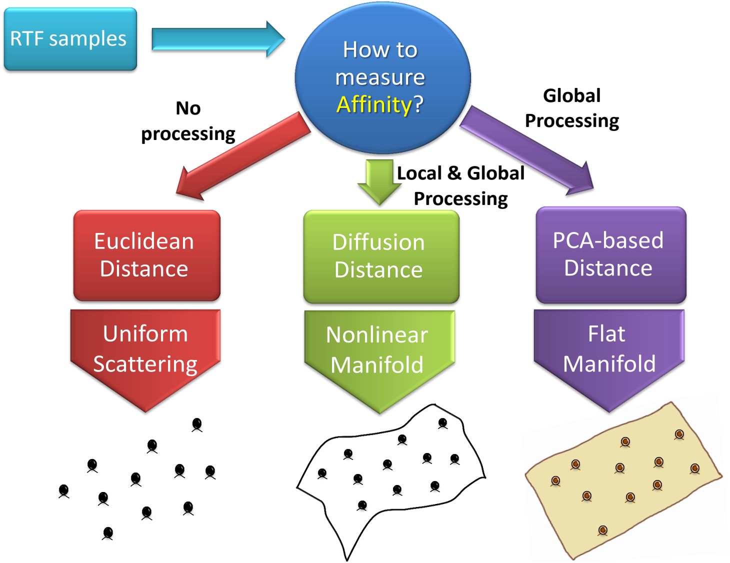


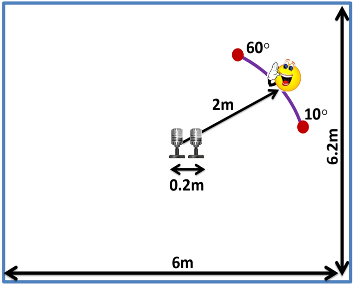
Which of the distance measures is proper?
What is the true underlying structure of the RTFs?

Simulation Results

Room setup

Simulate a reverberant room using the image method [Allen and Berkley, 1979]:

- Room dimension $6 \times 6.2 \times 3$m
- Microphones at: $[3, 3, 1]$ and $[3.2, 3, 1]$
- The source is positioned at 2m from the mics, the azimuth angle in $10^\circ \div 60^\circ$
- $T_{60} = 150/300/500$ ms
- SNR$=$ 20 dB

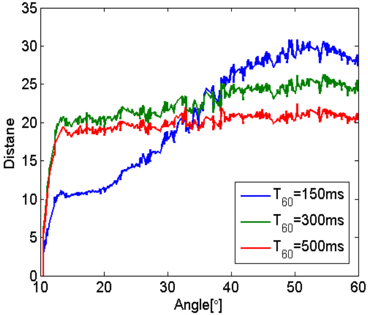


Test

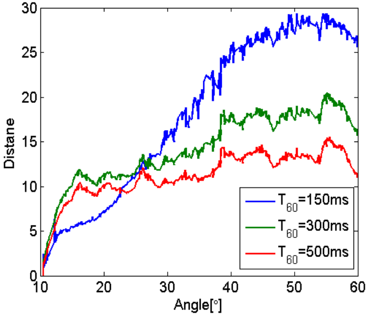
Measure the distance between each of the RTFs and the RTF corresponding to 10°:

- If monotonic with respect to the angle - proper distance
- If not monotonic with respect to the angle - improper distance

Euclidean Distance & PCA-based Distance [Laufer-Goldshtein et al., 2015]



(a) Euclidean Distance

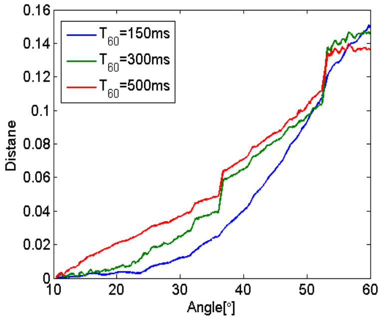


(b) PCA-based Distance

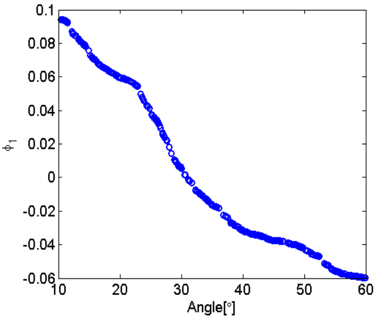
For both distance measures:

- Monotonic with respect to the angle only in a limited region
- This region becomes smaller as the reverberation time increases
- They are inappropriate for measuring angles' proximity

Diffusion Maps



(c) Diffusion Distance



(d) Diffusion Mapping

The diffusion distance:

- Monotonic with respect to the angle for almost the entire range
- It is an appropriate distance measure in terms of the source DOA
- Mapping corresponds well with angles - recovers the latent parameter

Outline

Semi-Supervised Approaches for Localization

Intermediate summary

- We have established the existence of an acoustic manifold in a specific environment
- The RTF was shown to be a proper feature vector that can capture the acoustic variability as a function of the source position (alternative feature vectors can be used)
- A brief introduction to manifold learning - a systematic methodology to infer the low-dimensional intrinsic controlling parameters of the data

Semi-Supervised Approaches for Localization (cont.)

What's next?

- Learning paradigms:
  1. Unsupervised localization ⇒ array constellation required (microphones positions or microphone inter-distance for DOA-only)
  2. Supervised localization ⇒ many labels
  3. Semi-supervised ⇒ utilizes a small number of labelled data and a large number of unlabelled data; array constellation not required
- Utilize the acoustic manifold to derive two data-driven approaches for speaker localization:
  1. Diffusion Distance Search (DDS) [Talmon et al., 2011, Laufer-Goldshtein et al., 2013]
  2. Manifold Regularization for Localization (MRL) [Laufer-Goldshtein et al., 2016b]

Goal: Recover the function f which transforms an RTF to position

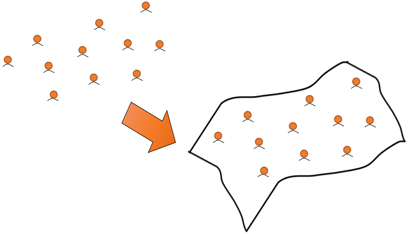
Semi-Supervised Approaches for Localization (cont.)

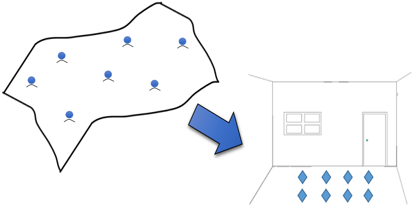
Unlabelled Samples

Labelled Samples

Recover the Manifold Structure

Anchor Points – Translate RTFs to Positions





Semi-Supervised Learning

Mixed of supervised (attached with known locations as anchors) and unsupervised (unknown locations) learning

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Why using unlabeled data?

1. Localization - training should fit the specific environment of interest:
   - Cannot generate a general database for all possible acoustic scenarios
   - Generating a large amount of labelled data is cumbersome/impractical
   - Unlabelled data is freely available - whenever someone is speaking

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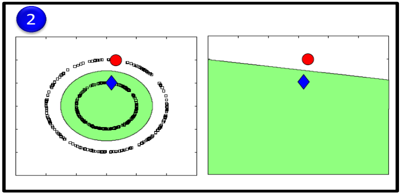
   - Unlabelled data is freely available - whenever someone is speaking
2. Unlabelled data can be utilize to recover the manifold structure

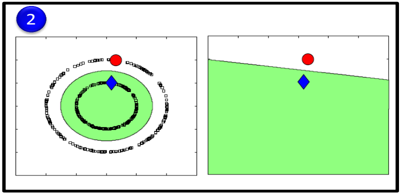


Semi-Supervised Learning

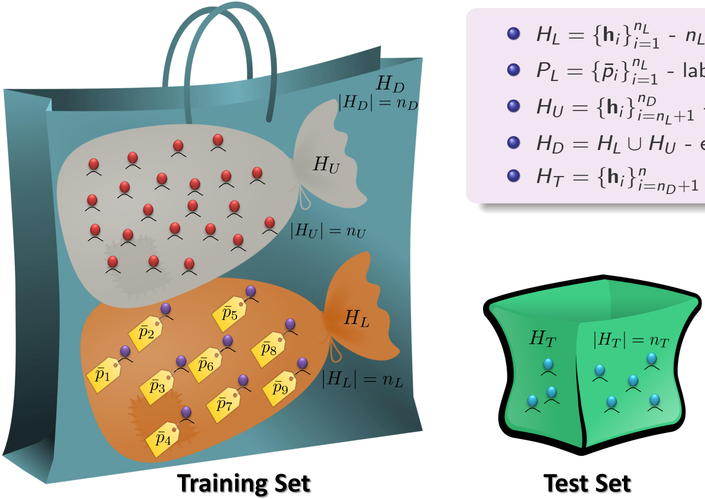
Mixed of supervised (attached with known locations as anchors) and unsupervised (unknown locations) learning

Why using unlabeled data?

1. Localization - training should fit the specific environment of interest:
   - Cannot generate a general database for all possible acoustic scenarios
   - Generating a large amount of labelled data is cumbersome/impractical
   - Unlabelled data is freely available - whenever someone is speaking
2. Unlabelled data can be utilize to recover the manifold structure
3. Semi-supervised learning is the natural setting for human learning



Datasets



- $H_L = \{\mathbf{h}_i\}_{i=1}^{n_L}$ - n_L labelled samples
- $P_L = \{\bar{p}_i\}_{i=1}^{n_L}$ - labels/positions
- $H_U = \{\mathbf{h}_i\}_{i=n_L+1}^{n_D}$ - n_U unlabelled samples
- $H_D = H_L \cup H_U$ - entire training set
- $H_T = \{\mathbf{h}_i\}_{i=n_D+1}^{n}$ - n_T test samples

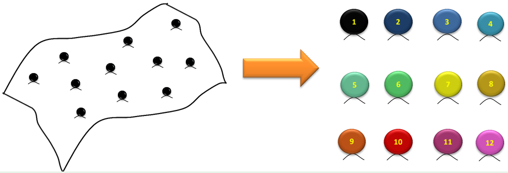
Diffusion Distance Search (DDS) [Talmon et al., 2011, Laufer-Goldshtein et al., 2013]

Diffusion mapping

- Construct $\mathbf{K}$, and normalise to obtain $\mathbf{P}$
- Employ EVD to obtain $\{\lambda_j, \varphi_j\}$
- Construct the map $\mathbf{\Phi}_d$:

$$\mathbf{\Phi}_d : \mathbf{h}_i \mapsto \left[\lambda_1 \varphi_1^{(i)}, \ldots, \lambda_d \varphi_d^{(i)}\right]^T$$

- Define diffusion distance: $D_{\mathrm{Diff}}(\mathbf{h}_\mathrm{l}, \mathbf{h}_i) = \|\mathbf{\Phi}_d(\mathbf{h}_l) - \mathbf{\Phi}_d(\mathbf{h}_i)\|_2$



How to incorporate new test samples $\mathbf{h}_\mathrm{t}$?

Diffusion Distance Search (DDS)

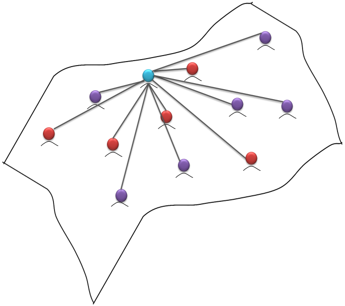
Nyström extension:

- Eigenvectors satisfy: $\lambda_j \boldsymbol{\varphi}_j = \mathbf{P}\boldsymbol{\varphi}_j$
- Can be written as:

$$\varphi_j^{(i)} = \frac{1}{\lambda_j} \sum_l p(\mathbf{h}_i, \mathbf{h}_l) \varphi_j^{(l)}$$

- For a new test point $\mathbf{h}_{\mathrm{t}}$:

$$\varphi_j^* = \frac{1}{\lambda_j} \sum_l p(\mathbf{h}_{\mathrm{t}}, \mathbf{h}_l) \varphi_j^{(l)}$$



Extension of the model for new $\mathbf{h}_{\mathrm{t}}$ (summary):

- Construct a nonsymmetric affinity vector $\mathbf{b}$: $b^{(l)} = p(\mathbf{h}_t, \mathbf{h}_l)$
- Apply Nyström extension:

$$\varphi_j^* = \frac{1}{\lambda_j} \mathbf{b}^T \boldsymbol{\varphi}_j \quad j \in \{1, \ldots, d\}$$

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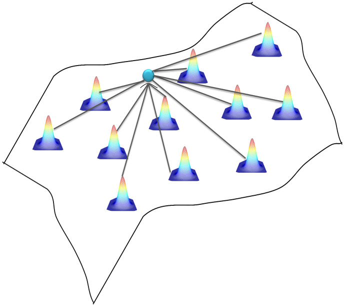

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$$\varphi_j^{*} = \frac{1}{\lambda_j} \mathbf{b}^T \varphi_j \quad j \in \{1, \ldots, d\}$$

Diffusion Distance Search (DDS)

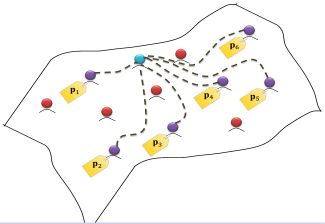
Localization:

Heuristic estimation: a linear combination of the labelled set positions according to kernelized diffusion distances:

$$\hat{p} = \sum_{i=1}^{n_L} \gamma\left(\mathbf{h}_i\right) p_i$$

where the weights $\gamma\left(\mathbf{h}_i\right)$ are given by:

$$\gamma\left(\mathbf{h}_i\right) = \frac{\exp\left\{-D_{\mathrm{Diff}}\left(\mathbf{h}_{\mathrm{t}}, \mathbf{h}_i\right)/\varepsilon_\gamma\right\}}{\sum_{j=1}^{l} \exp\left\{-D_{\mathrm{Diff}}\left(\mathbf{h}_{\mathrm{t}}, \mathbf{h}_j\right)/\varepsilon_\gamma\right\}}$$



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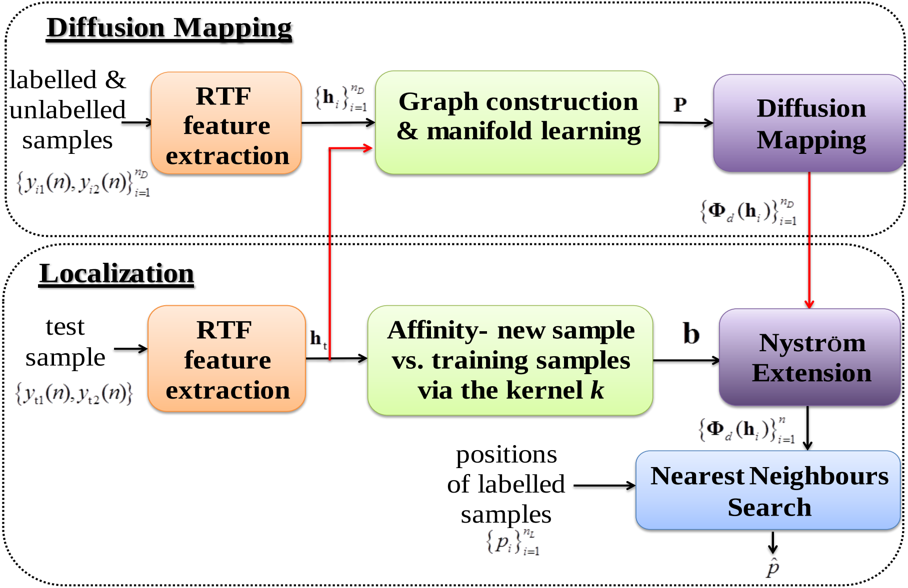
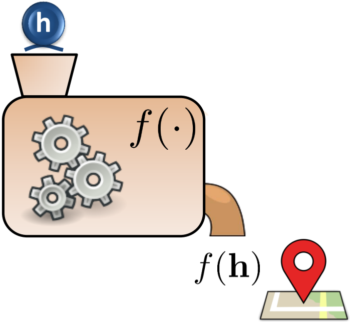


Manifold Regularization for Localization [Laufer-Goldshtein et al., 2016b]

Goal: Recover the function f which transforms an RTF to position



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Complex nonlinear relation between RTFs and positions

Infinite search space

How to prevent overfitting?

How to utilize unlabelled data?

Manifold Regularization for Localization [Laufer-Goldshtein et al., 2016b]

Goal: Recover the function f which transforms an RTF to position

Complex nonlinear relation between RTFs and positions

- Learn a data-driven model from training data

Infinite search space

- Work in a reproducing kernel Hilbert space (RKHS)

How to prevent overfitting?

- Add regularizations to control smoothness

How to utilize unlabelled data?

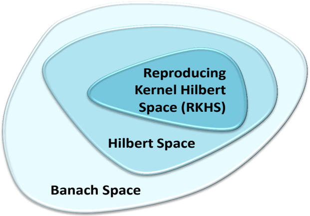
- Use manifold regularization

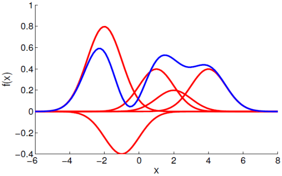
Reproducing Kernel Hilbert Space (RKHS) [Berlinet and Thomas-Agnan, 2011]

Moore-Aronszajn theorem: [Aronszajn, 1950]

For a positive definite kernel k on $\mathcal{M}$, there is a Hilbert space $\mathcal{H}_k$ (reproducing kernel Hilbert space, (RKHS)) that consists of functions on $\mathcal{M}$, satisfying:

- $k(\mathbf{h}, \cdot) \in \mathcal{H}_k, \forall \mathbf{h} \in \mathcal{M}$;
- $\mathrm{span}\{k(\mathbf{h}, \cdot); \mathbf{h} \in \mathcal{M}\}$ is dense in $\mathcal{H}_k$;
- **The reproducing property:** $\langle f(\cdot), k(\mathbf{h}, \cdot)\rangle = f(\mathbf{h}), \forall f \in \mathcal{H}_k, \mathbf{h} \in \mathcal{M}$.





Optimization and Manifold Regularization

Optimization in a reproducing kernel Hilbert space (RKHS) [Belkin et al., 2006]:

$$f^* = \underset{f \in \mathcal{H}_k}{\operatorname{argmin}} \frac{1}{n_L} \sum_{i=1}^{n_L} (\bar{p}_i - f(\mathbf{h}_i))^2 + \gamma_k \|f\|_{\mathcal{H}_k}^2 + \gamma_M \|f\|_{\mathcal{M}}^2$$

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Cost function

$\frac{1}{n_L} \sum_{i=1}^{n_L} (\bar{p}_i - f(\mathbf{h}_i))^2$

correspondence between function values and labels

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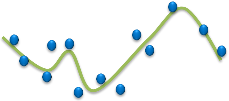
Tikhonov Regularization

$\|f\|^2_{\mathcal{H}_k}$

smoothness condition in the RKHS



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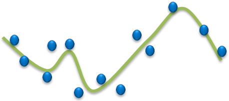
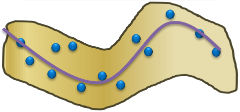


Manifold Regularization

$\|f\|^2_{\mathcal{M}}$

smoothness penalty with respect to the manifold



Manifold Regularization

Smoothness on the manifold: A reminder

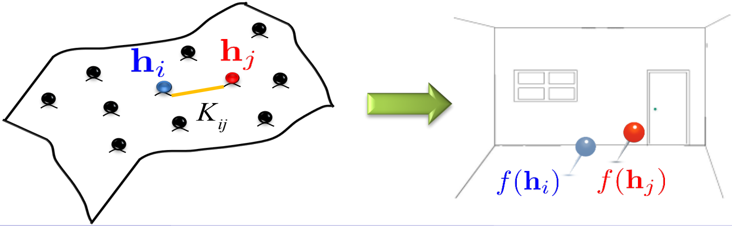
- The graph Laplacian:

$$\mathbf{L} = \mathbf{D} - \mathbf{K}$$

- Define the manifold regularization by:

$$\|f\|_{\mathcal{M}}^2 = \mathbf{f}_D^T \mathbf{L} \mathbf{f}_D = \frac{1}{2} \sum_{i,j=1}^{n_D} K_{ij} \left(f(\mathbf{h}_i) - f(\mathbf{h}_j) \right)^2$$

$\mathbf{f}_D^T = [f_1, f_2, \ldots, f_{n_D}]$ comprising labelled and unlabelled training data



Optimization and Manifold Regularization

The optimization problem can be recast as:

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The Representer theorem: [Schölkopf et al., 2001]

The minimizer over $\mathcal{H}_k$ of the regularized optimization is represented by:

$$f^*(\mathbf{h}) = \sum_{i=1}^{n_D} a_i k(\mathbf{h}_i, \mathbf{h})$$

where $k : \mathcal{M} \times \mathcal{M} \rightarrow \mathbb{R}$ is the reproducing kernel of $\mathcal{H}_k$
with $K_{ij} = k(\mathbf{h}_i, \mathbf{h}_j) = \exp\left\{-\frac{\|\mathbf{h}_i - \mathbf{h}_j\|^2}{\varepsilon}\right\}$

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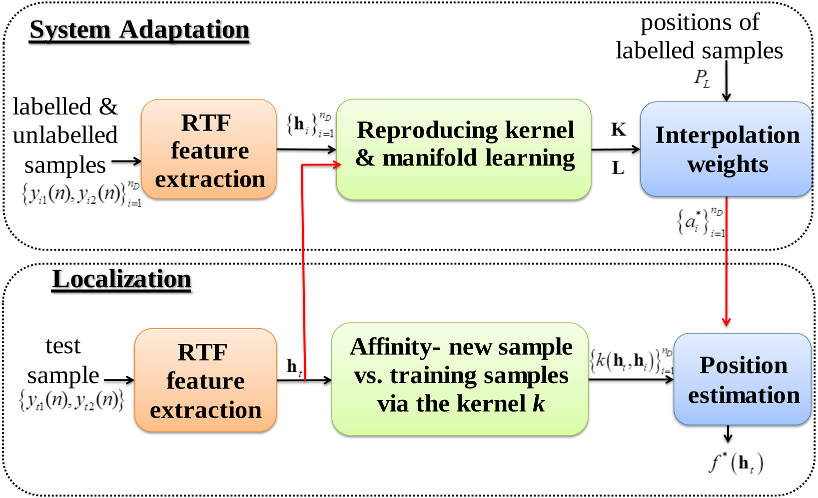
Manifold Regularization for Localization (MRL)

[Laufer-Goldshtein et al., 2017]



$K_{ij} = k(\mathbf{h}_i, \mathbf{h}_j)$

Simulation Results

Setup:

- **Source positions:** angles between $10^\circ \div 60^\circ$
- **Training:** 6 labelled, 400 unlabelled (SNR=10 dB)

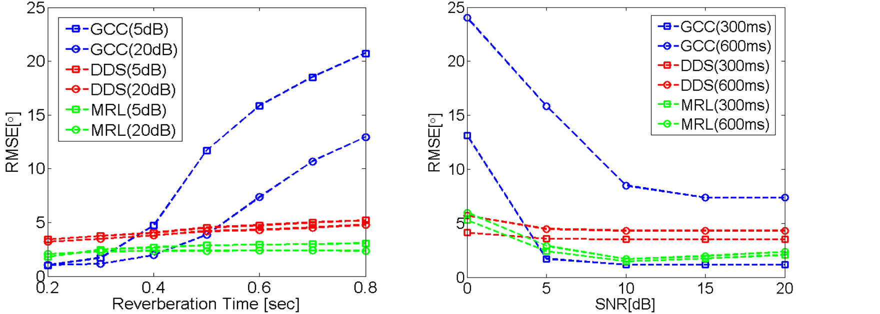


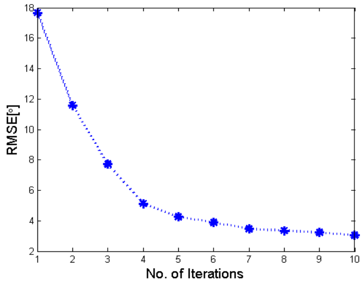
Figure: RMSEs of GCC, DDS and MRL as a function of reverberation time (left), SNR (right)

MRL achieves 2° accuracy in typical noisy and reverberant environments

Simulation Results - MRL

Iterative simulation:

- Source positions: angles between $0^\circ \div 180^\circ$
- Start with 19 labelled samples
- Each iteration add 80 unlabelled samples
- $T_{60} = 500$ ms and SNR=20 dB

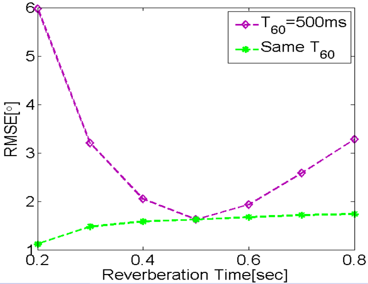


Sensitivity to reverberation level:

- Train with a fixed reverberation time of 500 ms.

→ for small mismatch - small increase in error level

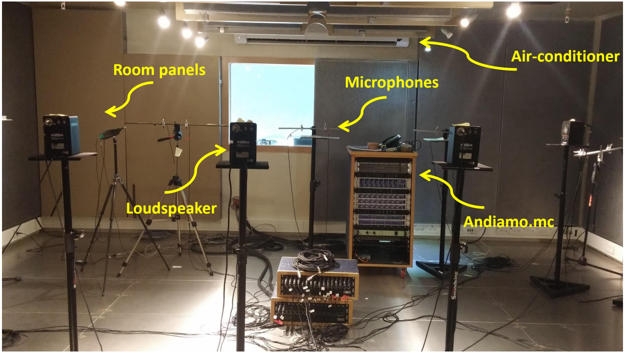
→ for large mismatch - large increase in error level



Recordings setup

Setup:

- Real recordings carried out at Bar-Ilan acoustic lab
- A $6 \times 6 \times 2.4$m room controllable reverberation time (set to 620ms)
- Region of interest: a 4m long line at 2.5m distance from the mics



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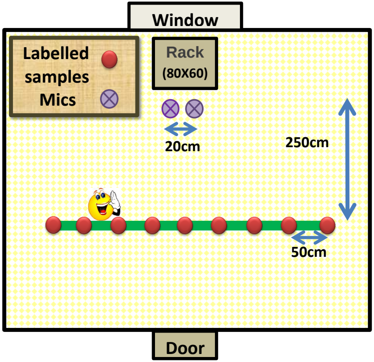


Experimental Results

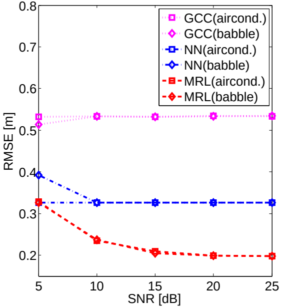
[Laufer-Goldshtein et al., 2016b]

Setup:

- Training: 5 labelled samples (1m resolution), 75 unlabelled samples
- Test: 30 random samples in the defined region
- Two noise types: air-conditioner noise and babble noise

Compare with:

- Nearest-neighbour (NN)
- Generalized cross-correlation (GCC) method [Knapp and Carter, 1976]



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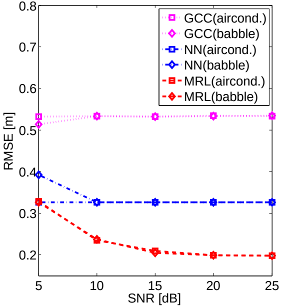
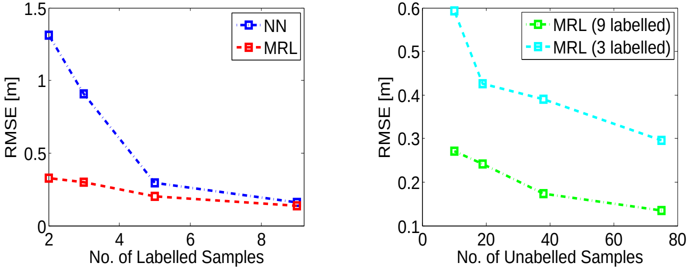


The MRL algorithm outperforms the two other methods

Effect of Labelled & Unlabelled Samples

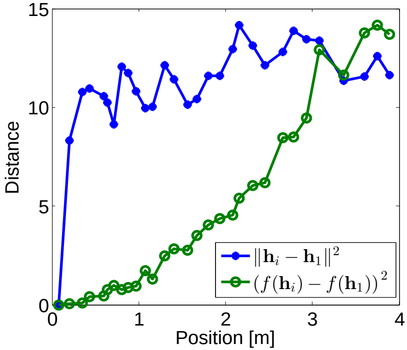


Effect of increasing the amount of labelled/unlabelled samples

- → As the size of the labelled set is reduced - performance gap increases
- → Locate the source even with few labelled samples, using unlabelled information

Why does Nearest-Neighbour Fail?

Compare distances before and after mapping



Monotony/Order

- Before mapping - monotonic/ordered only in a limited region
- After mapping - monotonic/ordered for almost the entire range

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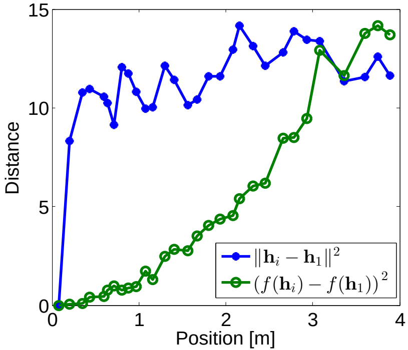

Monotony/Order

- Before mapping - monotonic/ordered only in a limited region
- After mapping - monotonic/ordered for almost the entire range

We conclude:

$\rightarrow$ RTFs lie on a nonlinear manifold - linear only for small patches

$\rightarrow$ NN ignores the manifold, MRL exploits the manifold structure

Localization on Manifolds

Two Data-Driven Localization Algorithms

Diffusion Distance Search (DDS)

Diffusion Embedding

Neighbors Search in the Embedded Space

1. Two stage
2. Better than GCC

Diffusion Maps

Manifold Regularization for Localization (MRL)

Manifold Regularization

Regularized Optimization

Optimization in RKHS

1. One stage
2. Best performance

Outline

Manifold-Based Bayesian Inference [Laufer-Goldshtein et al., 2016a]

Estimate the function f which transforms an RTF to position using a Bayesian approach with a data-driven geometric model

Update Belief

$$\underbrace{p(f|P_L, H_L)}_{\text{Posterior}} \propto \underbrace{p(P_L|f, H_L)}_{\text{Likelihood}} \cdot \underbrace{p(f|H_L)}_{\text{Prior}}$$

Posterior

Likelihood

Correspondence between the function values and the labels

Prior

A priori belief on the behavior of f - control smoothness

Manifold-Based Bayesian Inference [Laufer-Goldshtein et al., 2016a]

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Likelihood: Correspondence between the function values and the labels

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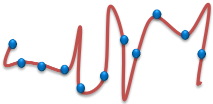

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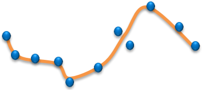

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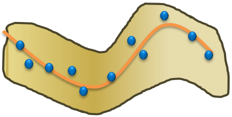
Posterior

Likelihood

Correspondence between the function values and the labels

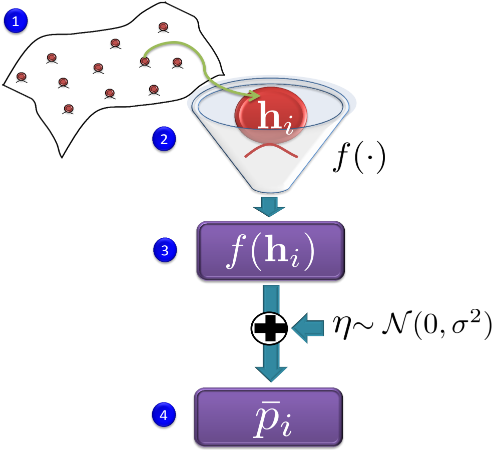
Manifold-Based Prior

a priori belief on the properties of f - smoothness with respect to the manifold



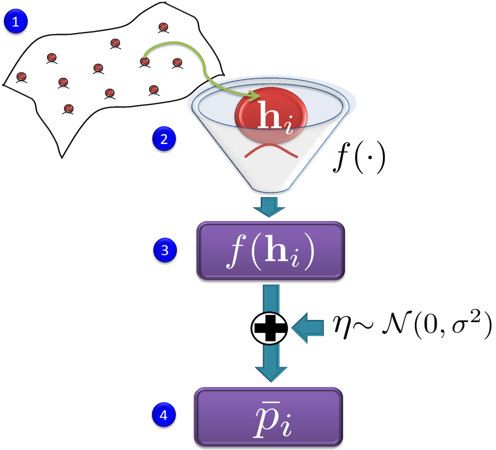
The Likelihood Function

1. An RTF is sampled from the manifold $\mathcal{M}$
2. The function f follows a stochastic process
3. The function receives an RTF sample and returns the position
4. Measure a noisy position due to imperfect calibration



The Likelihood Function

1. An RTF is sampled from the manifold $\mathcal{M}$
2. The function f follows a stochastic process
3. The function receives an RTF sample and returns the position
4. Measure a noisy position due to imperfect calibration



→ Likelihood function: $p(P_L|f,H_L) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n_L}(\bar{p}_i - f(\mathbf{h}_i))^2\right\}$

Standard Prior Probability

Standard Gaussian process [Rasmussen and Williams, 2006]:

- The function f follows a Gaussian process:

$$f(\mathbf{h}) \sim \mathcal{GP}\left(\nu(\mathbf{h}), k(\mathbf{h}, \mathbf{h}_i)\right)$$

- ν is the mean function (choose $\nu \equiv 0$)
- k is the covariance function.
- The r.v. $\mathbf{f}_H = [f(\mathbf{h}_1), \dots, f(\mathbf{h}_n)]$ has a joint Gaussian distribution:

$$\mathbf{f}_H \sim \mathcal{N}(\mathbf{0}_n, \mathbf{\Sigma}_{HH})$$

  where $\mathbf{\Sigma}_{HH}$ is the covariance matrix with elements $k(\mathbf{h}_i, \mathbf{h}_j)$
- Common choice: a Gaussian kernel $k(\mathbf{h}_i, \mathbf{h}_j) = \exp\{-\|\mathbf{h}_i - \mathbf{h}_j\|^2/\varepsilon_k\}$

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✗ The correlation for intermediate distances may be incorrectly assessed

✗ Does not exploit the available set of unlabelled data H_U

Manifold-Based Prior Probability [Sindhwani et al., 2007]

Discretization of the manifold

- The manifold is empirically represented by a graph G, with weights:

$$W_{ij} = \begin{cases} \exp\left\{-\frac{\|\mathbf{h}_i - \mathbf{h}_j\|^2}{\varepsilon_w}\right\} & \text{if } \mathbf{h}_j \in \mathcal{N}_i \text{ or } \mathbf{h}_i \in \mathcal{N}_j \\ 0 & \text{otherwise} \end{cases}$$

- The graph Laplacian of G: $\mathbf{L} = \mathbf{D} - \mathbf{W}$, where $D_{ii} = \sum_{j=1}^{n} W_{ij}$.

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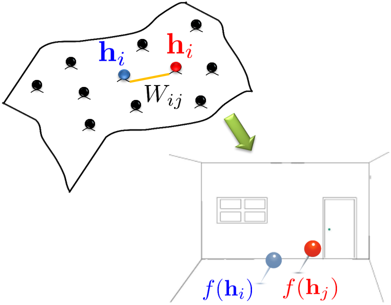
Statistical formulation

- Geometry variables $\mathcal{G}$ – represent the manifold structure
- The likelihood of $\mathcal{G}$:

$$P(\mathcal{G}|\mathbf{f}_D) \propto \exp\left\{-\frac{\gamma_M}{2}\left(\mathbf{f}_D^T \mathbf{L} \mathbf{f}_D\right)\right\}$$

- We showed (based on all n_D training samples):

$$\mathbf{f}_D^T \mathbf{L} \mathbf{f}_D = \frac{1}{2} \sum_{i,j=1}^{n_D} W_{ij} \left(f(\mathbf{h}_i) - f(\mathbf{h}_j)\right)^2$$



Manifold-Based Prior Probability [Sindhwani et al., 2007]

Manifold-Based GP Prior — **Likelihood of Geometry Variables** — **Standard GP Prior**

$$p(\mathbf{f}_H|\mathcal{G}) \propto p(\mathcal{G}|\mathbf{f}_H) \cdot p(\mathbf{f}_H)$$

The covariance is formed by a manifold-based kernel $\tilde{k}$

correspondence between the function values and the manifold structure

The covariance is formed by a standard kernel k

Assume: $p(\mathcal{G}|\mathbf{f}_H) \propto p(\mathcal{G}|\mathbf{f}_D)$

$$p(\mathbf{f}_H|\mathcal{G}) = \mathcal{N}(\mathbf{0}_m, \tilde{\mathbf{\Sigma}}_{HH})$$

$$P(\mathcal{G}|\mathbf{f}_D) \propto \exp\left\{ - \frac{\gamma_M}{2}\left(\mathbf{f}_D^T \mathbf{M} \mathbf{f}_D\right)\right\}$$

$$p(\mathbf{f}_H) = \mathcal{N}(\mathbf{0}_m, \mathbf{\Sigma}_{HH})$$

$$\tilde{\Sigma}_{\mathrm{HH}} \Leftrightarrow \tilde{k}(\mathbf{h}_i, \mathbf{h}_j)$$

Manifold-Based Bayesian Inference [Laufer-Goldshtein et al., 2016a]

$$f^* = \operatorname{argmin}_{f \in \mathcal{H}_k} \frac{1}{n_L} \sum_{i=1}^{n_L} (\bar{p}_i - f(\mathbf{h}_i))^2 + \gamma_k \|f\|^2_{\mathcal{H}_k} + \gamma_M \mathbf{f}_D^T \mathbf{M} \mathbf{f}_D$$

Search in RKHS defined by the kernel k

Cost Function

$\mathcal{H}_k$ **norm**

Manifold Regularization

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Search in RKHS defined by the kernel k | **Cost Function** | **$\mathcal{H}_k$ norm** | **Manifold Regularization**

$$f^* = \operatorname{argmin}_{f \in \mathcal{H}_{\tilde{k}}} \frac{1}{n_L} \sum_{i=1}^{n_L} (\bar{p}_i - f(\mathbf{h}_i))^2 + \gamma_k \|f\|^2_{\mathcal{H}_{\tilde{k}}}$$

Search in RKHS defined by the kernel $\tilde{k}$ | **Cost Function** | **$\mathcal{H}_{\tilde{k}}$ norm**

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Search in RKHS defined by the kernel $\tilde{k}$

$$\underbrace{p(f|P_L, H_L, H_U)}_{\text{Posterior}} \propto \underbrace{p(P_L|f, H_L)}_{\text{Likelihood Function}} \cdot \underbrace{p(f|H_L, H_U)}_{\text{Manifold-Based Prior}}$$

f is a Gaussian Process with Covariance $\tilde{k}$

Bayesian Localization

Joint probability:

- Goal: estimate the function value at some test sample $\mathbf{h}_t \in \mathcal{M}$
- The training positions $\bar{\mathbf{p}}_L = \mathrm{vec}\{P_L\}$ and $f(\mathbf{h}_t)$ are jointly Gaussian:

$$\begin{bmatrix} \bar{\mathbf{p}}_L \\ f(\mathbf{h}_t) \end{bmatrix} \Bigg| H_L, H_U \sim \mathcal{N}\left(\mathbf{0}_{n_L+1}, \begin{bmatrix} \tilde{\boldsymbol{\Sigma}}_{LL} + \sigma^2 \mathbf{I}_{n_L} & \tilde{\boldsymbol{\Sigma}}_{Lt} \\ \tilde{\boldsymbol{\Sigma}}_{Lt}^T & \tilde{\boldsymbol{\Sigma}}_{tt} \end{bmatrix}\right)$$

- The elements of $\tilde{\boldsymbol{\Sigma}}_{LL}$, $\tilde{\boldsymbol{\Sigma}}_{Lt}$ and $\tilde{\boldsymbol{\Sigma}}_{tt}$ are calculated by the manifold-regularized kernel

$$\mathrm{cov}(f(\mathbf{h}_r), f(\mathbf{h}_l)) \equiv \tilde{k}(\mathbf{h}_r, \mathbf{h}_l)$$

- Note that the unlabelled points are implicitly considered in the covariance terms

Bayesian Localization (cont.)

MAP/MMSE estimator:

- The posterior

$$p(f(\mathbf{h}_t)|P_L, H_L, H_U) \sim \mathcal{N}(\hat{f}(\mathbf{h}_t), \mathrm{var}(\hat{f}(\mathbf{h}_t)))$$

is a multivariate Gaussian, where:

  - The MAP/MMSE estimator of $f(\mathbf{h}_t)$ is given by:

$$\hat{f}(\mathbf{h}_t) = \tilde{\mathbf{\Sigma}}_{Lt}^T \left(\tilde{\mathbf{\Sigma}}_{LL} + \sigma^2 \mathbf{I}_{n_L}\right)^{-1} \bar{\mathbf{p}}_L$$

  - The estimation confidence:

$$\mathrm{var}(\hat{f}(\mathbf{h}_t)) = \tilde{\Sigma}_{tt} - \tilde{\mathbf{\Sigma}}_{Lt}^T \left(\tilde{\mathbf{\Sigma}}_{LL} + \sigma^2 \mathbf{I}_{n_L}\right)^{-1} \tilde{\mathbf{\Sigma}}_{Lt}$$

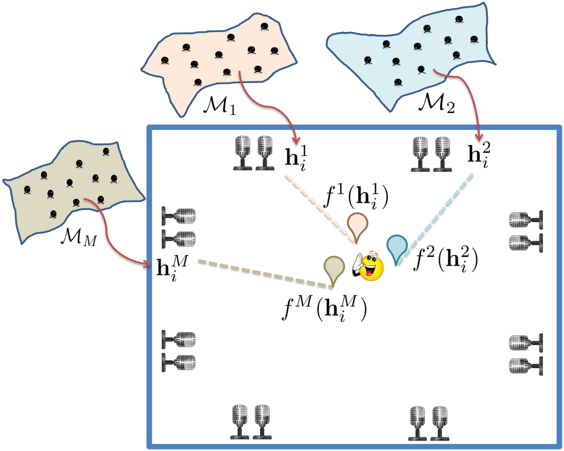
Learning the Hyperparameters: [Laufer-Goldshtein et al., 2017]

- The hyperparameters:
  - Kernel scales ϵ
  - Weights γ (Gaussian process variance)

  can be inferred from the data by optimizing the likelihood function of the labelled samples

Outline

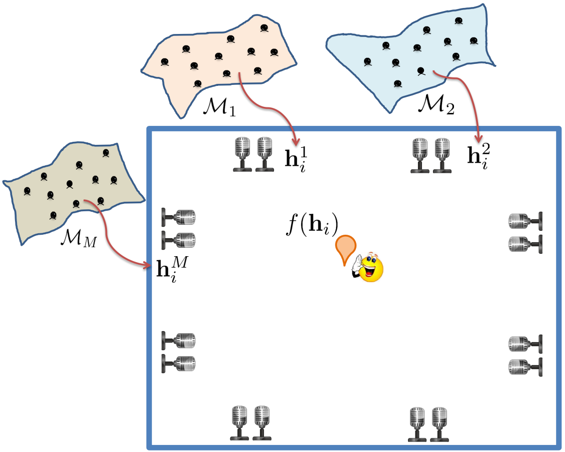
Source Localization with Ad Hoc Array [Laufer-Goldshtein et al., 2017]



Each node

- Represents a different view point on the same acoustic event
- Induces relations between RTFs according to the associated manifold

Source Localization with Ad Hoc Array [Laufer-Goldshtein et al., 2017]

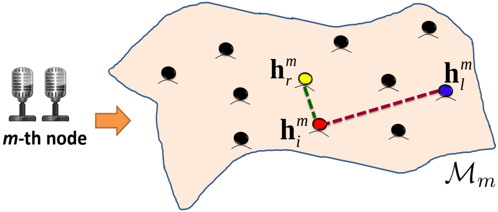


$$\mathbf{h}_i = [(\mathbf{h}_i^1)^T, (\mathbf{h}_i^2)^T, \ldots, (\mathbf{h}_i^M)^T]^T \in \cup_{m=1}^M \mathcal{M}_m$$

How to fuse the different views in a unified mapping $f : \cup_{m=1}^M \mathcal{M}_m \mapsto \mathbb{R}$?

Inta-Manifold Relations

The mapping follows a Gaussian process $f^m(\mathbf{h}^m) \sim \mathcal{GP}(0, \tilde{k}_m(\mathbf{h}^m, \mathbf{h}_i^m))$



Covariance function

Defined by a new manifold-based covariance function:

$$\mathrm{cov}\left(f^m(\mathbf{h}_r^m), f^m(\mathbf{h}_l^m)\right) \equiv \tilde{k}_m(\mathbf{h}_r^m, \mathbf{h}_l^m) = \sum_{i=1}^{n_D} k_m(\mathbf{h}_r^m, \mathbf{h}_i^m) k_m(\mathbf{h}_l^m, \mathbf{h}_i^m)$$

$$= 2k_m(\mathbf{h}_r^m, \mathbf{h}_l^m) + \sum_{\substack{i=1 \\ i \neq l,r}}^{n_D} k_m(\mathbf{h}_r^m, \mathbf{h}_i^m) k_m(\mathbf{h}_l^m, \mathbf{h}_i^m)$$

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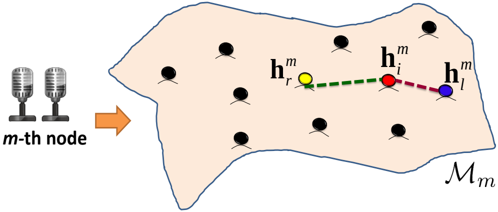

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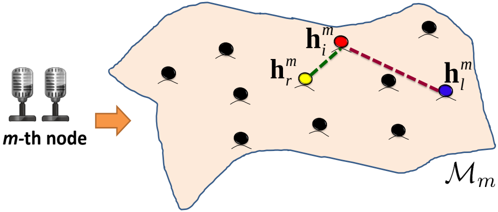

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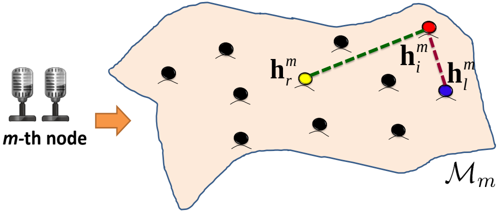

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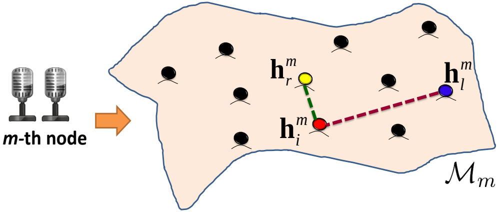

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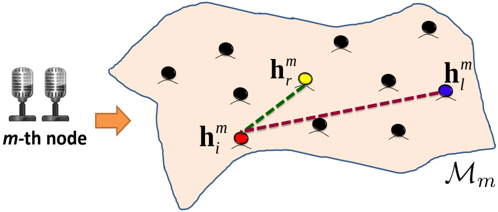

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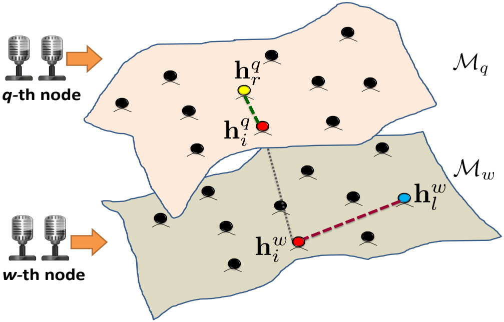
Inter-Manifold Relations

How to measure relations between RTFs from different nodes?



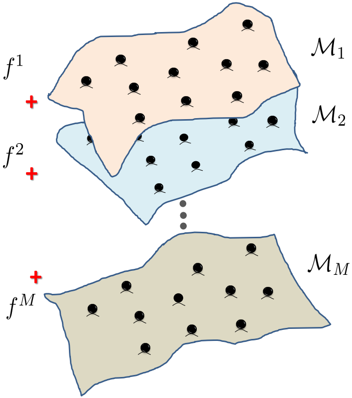
Multi-node covariance

The covariance between $f^q(\mathbf{h}_r^q)$ and $f^w(\mathbf{h}_r^w)$:

$$\mathrm{cov}\left(f^q(\mathbf{h}_r^q), f^w(\mathbf{h}_r^w)\right) = \sum_{i=1}^{n_D} k_q(\mathbf{h}_r^q, \mathbf{h}_i^q) k_w(\mathbf{h}_l^w, \mathbf{h}_i^w)$$

Multiple Manifold Gaussian Process (MMGP)

Define the average process $f = \frac{1}{M}(f^1 + f^2 + \ldots + f^M) \sim \mathcal{GP}(0, \tilde{k})$



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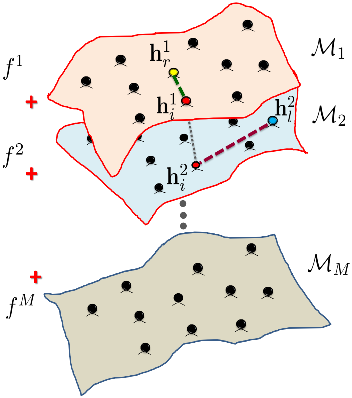

The covariance between $f(\mathbf{h}_r)$ and $f(\mathbf{h}_l)$

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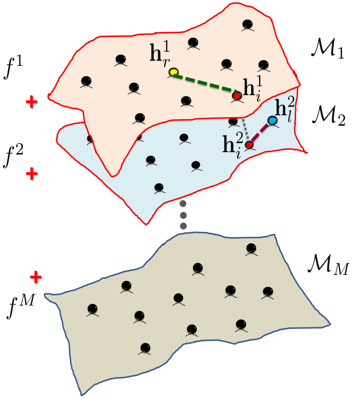

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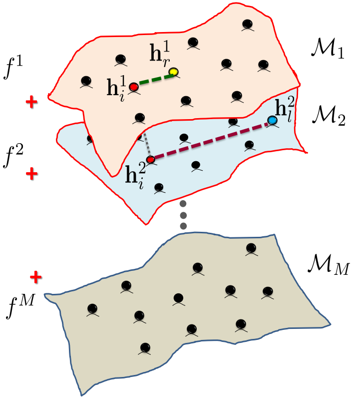

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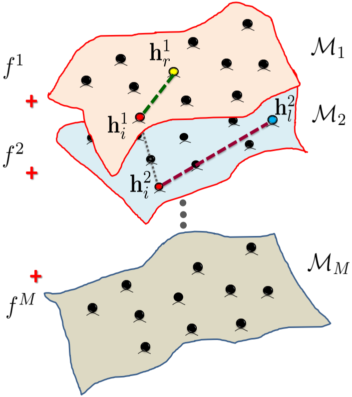

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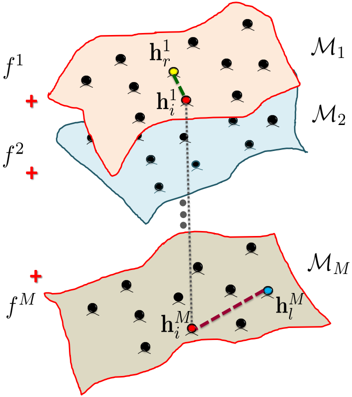

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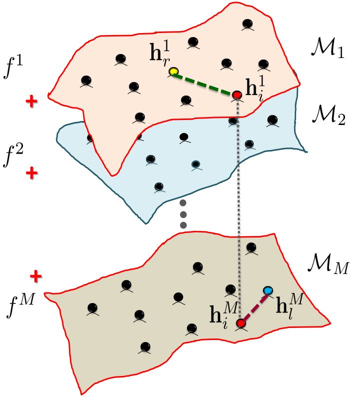

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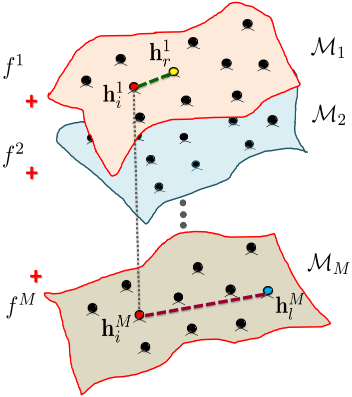

The covariance between $f(\mathbf{h}_r)$ and $f(\mathbf{h}_l)$

$$\mathrm{cov}(f(\mathbf{h}_r), f(\mathbf{h}_l)) \equiv \tilde{k}(\mathbf{h}_r, \mathbf{h}_l) = \frac{1}{M^2} \sum_{q,w=1}^{M} \sum_{i=1}^{n_D} k_q(\mathbf{h}_r^q, \mathbf{h}_i^q) k_w(\mathbf{h}_l^w, \mathbf{h}_i^w)$$

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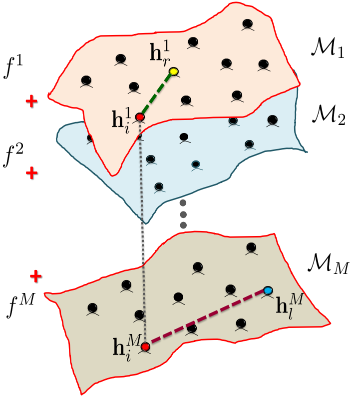

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Bayesian Multi-View Localization

Joint probability

- Goal: estimate the function value at some test sample $\mathbf{h}_t$
- The training positions $\bar{\mathbf{p}}_L = \mathrm{vec}\{P_L\}$ and $f(\mathbf{h}_t)$ are jointly Gaussian:

$$\begin{bmatrix} \bar{\mathbf{p}}_L \\ f(\mathbf{h}_t) \end{bmatrix} \Bigg| H_L, H_U \sim \mathcal{N}\left(\mathbf{0}_{n_L+1}, \begin{bmatrix} \tilde{\mathbf{\Sigma}}_{LL} + \sigma^2 \mathbf{I}_{n_L} & \tilde{\mathbf{\Sigma}}_{Lt} \\ \tilde{\mathbf{\Sigma}}_{Lt}^T & \tilde{\mathbf{\Sigma}}_{tt} \end{bmatrix}\right)$$

- The elements of $\tilde{\mathbf{\Sigma}}_{LL}$, $\tilde{\mathbf{\Sigma}}_{Lt}$ and $\tilde{\mathbf{\Sigma}}_{tt}$ are calculated by the multiple manifold kernel

$$\mathrm{cov}(f(\mathbf{h}_r), f(\mathbf{h}_l)) \equiv \tilde{k}(\mathbf{h}_r, \mathbf{h}_l)$$

- Note that the unlabelled points are implicitly considered in the covariance terms

Bayesian Multi-View Localization (cont.)

MAP/MMSE estimator:

- The posterior

$$p(f(\mathbf{h}_t)|P_L, H_L, H_U) \sim \mathcal{N}(\hat{f}(\mathbf{h}_t), \mathrm{var}(\hat{f}(\mathbf{h}_t)))$$

is a multivariate Gaussian, where:

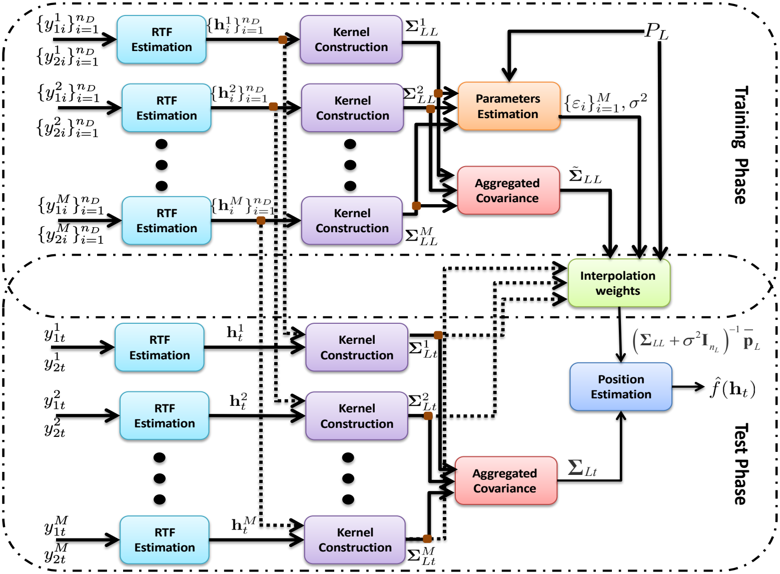
- The MAP/MMSE estimator of $f(\mathbf{h}_t)$ is given by:

$$\hat{f}(\mathbf{h}_t) = \tilde{\boldsymbol{\Sigma}}_{Lt}^T \left(\tilde{\boldsymbol{\Sigma}}_{LL} + \sigma^2 \mathbf{I}_{n_L} \right)^{-1} \bar{\mathbf{p}}_L$$

- The estimation confidence

$$\mathrm{var}(\hat{f}(\mathbf{h}_t)) = \tilde{\Sigma}_{tt} - \tilde{\boldsymbol{\Sigma}}_{Lt}^T \left(\tilde{\boldsymbol{\Sigma}}_{LL} + \sigma^2 \mathbf{I}_{n_L} \right)^{-1} \tilde{\boldsymbol{\Sigma}}_{Lt}$$

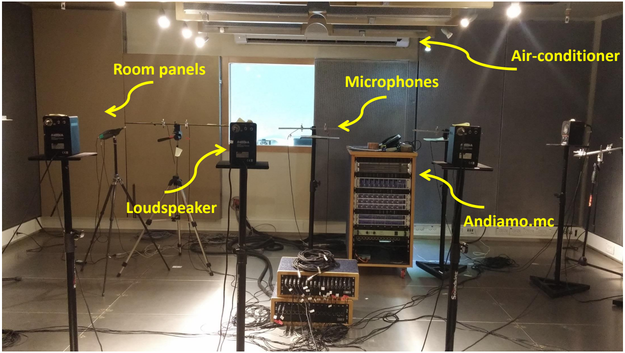
Multiple-Manifold Gaussian Process (MMGP)



Recordings Setup

Setup:

- Real recordings carried out at Bar-Ilan acoustic lab
- A $6 \times 6 \times 2.4$m room controllable reverberation time (set to 620ms)
- Region of interest: Source position is confined to a 2.8×2.1m area
- 3 microphone pairs with inter-distance of 0.2m



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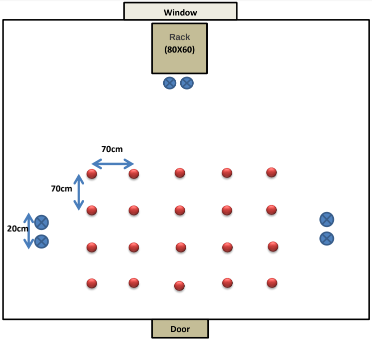


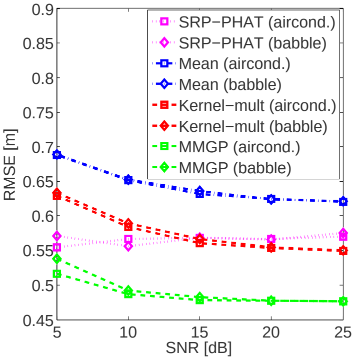
Experimental Results [Laufer-Goldshtein et al., 2017]

Setup:

- Training: 20 labelled samples (0.7m resolution), 50 unlabelled samples
- Test: 25 random samples in the defined region
- Two noise types: air-conditioner noise and babble noise

Compare with:

- Concatenated independent measurements (Kernel-mult)
- Average of single-node estimates (Mean)
- Beamformer scanning (SRP-PHAT [DiBiase et al., 2001])

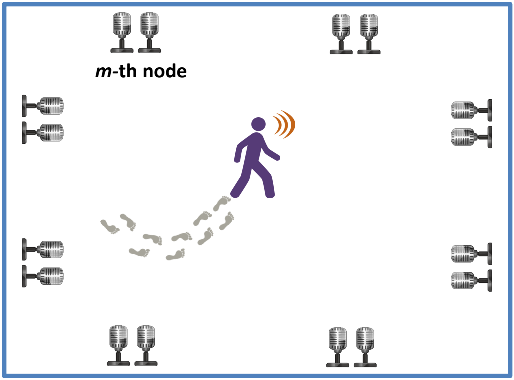


Outline

Speaker Tracking

Scenario:

- A source is moving in a reverberant enclosure
- Measured by an ad-hoc network with distributed microphones
- Microphones are arranged in M pairs - "nodes"



Bayesian Inference for Source Tracking

Standard state-space model

$$\mathbf{p}(t) = b(\mathbf{p}(t-1)) + \boldsymbol{\xi}(t)$$
$$\mathbf{q}(t) = c(\mathbf{p}(t)) + \zeta(t)$$

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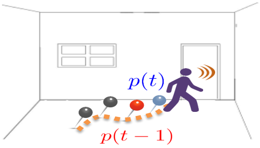
Propagation model

- Relates current and previous positions using random walk model or Langevin model
- Independent of measurements
- Noise statistic is unknown



Bayesian Inference for Source Tracking

Standard state-space model

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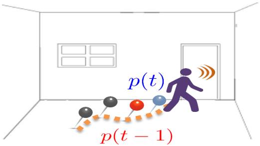
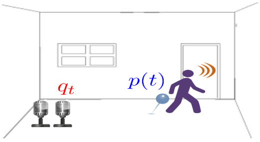
Propagation model

- Relates current and previous positions using random walk model or Langevin model
- Independent of measurements
- Noise statistic is unknown

Observation model

- Relates current position to measurements
- Examples: TDOA readings or SRP output
- Noise statistic is unknown





Data Model

Microphone signals:

The signal measured by the jth microphone in the mth node:

$$y^{mj}(t) = \sum_{\tau} a_t^{mj}(\tau) s(t-\tau) + u^{mj}(t), \quad 1 \leq m \leq M, \; j = 1, 2$$

- t - time index
- $s(t)$ - source signal
- a_t^{mj} - time-varying acoustic impulse response (AIR)
- $u^{mj}(t)$ - noise signal

Feature extraction:

- Use the RTF:

$$H^m(t,f) = \frac{A^{m2}(t,f)}{A^{m1}(t,f)}$$

- Represents the acoustic paths and is independent of the source signal

Time-Varying Relative Transfer Function (RTF)

- Instantaneous RTFs are estimated using the PSD and cross-PSD of the microphone signals at node m (low-noise):

$$\hat{H}_0^m(t,f) \simeq \frac{\hat{\Phi}_{21}^m(t,f)}{\hat{\Phi}_{11}^m(t,f)} = \frac{\sum_{n=t-L/2}^{t+L/2} Y^{m2}(n,f)Y^{m1*}(n,f)}{\sum_{n=t-L/2}^{t+L/2} Y^{m1}(n,f)Y^{m1*}(n,f)}$$

- Time-varying RTFs are estimated by recursive smoothing:

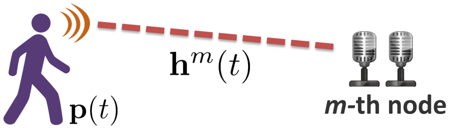
$$\hat{H}^m(t,f) = \gamma \hat{H}_0^m(t,f) + (1-\gamma)\hat{H}^m(t-1,f)$$

- Feature vectors are obtained by concatenating all relevant frequencies and all nodes:

$$\mathbf{h}^m(t) = \left[\hat{H}^m(t,f_1),\ldots,\hat{H}^m(t,f_F)\right]$$

$$\mathbf{h}(t) = \left[\mathbf{h}^{1T}(t),\ldots,\mathbf{h}^{MT}(t)\right]^T$$

Time-Varying Relative Transfer Function (RTF) (cont.)



- We assume the availability of n_L labelled RTFs with known positions:

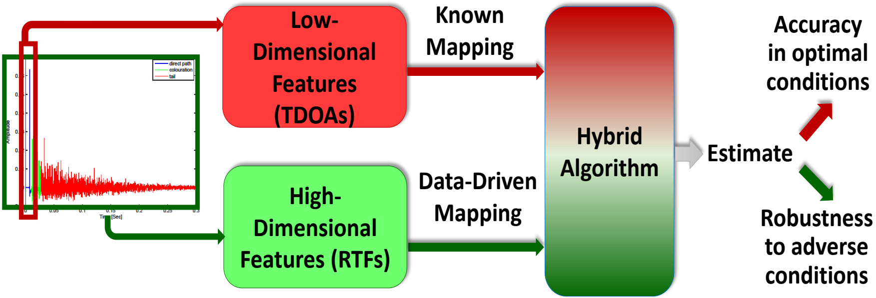
$$\{\mathbf{h}\}_{i=1}^{n_L} \Leftrightarrow \{\mathbf{p}\}_{i=1}^{n_L}$$

- These training RTFs can be estimated with static sources, hence a long observation interval L can be used and the recursive smoothing is not required

Hybrid Tracking [Laufer-Goldshtein et al., 2018b]

Combine TDOA-based approach with manifold-based approach:

- Manifold-based propagation model (non-arbitrary)
- TDOA-based observation model
- Combines Classical TDOA-based localization with the entire acoustic fingerprint



Derivation of the Manifold-Based Propagation Model

- Let $\mathbf{h}(t)$ be a test sample with unknown position $\mathbf{p}(t)$
- Define a subset of $N \leq n_L$ neighboring training samples $\{\mathbf{h}_{t_i}\}_{i=1}^{N}$:

$$\{\mathbf{h}_{t_i} | \|\mathbf{h}(t) - \mathbf{h}_{t_i}\| < \eta(N),\ i = 1, \ldots, N,\ t_i \in \{1, \ldots, n_L\}\}$$

  with $\eta(N)$ the neighborhood radius
- Let $\mathbf{f}_{t,c} = [f_c(\mathbf{h}(t)), f_c(\mathbf{h}_{t_1}), \ldots, f_c(\mathbf{h}_{t_N})]^T$ denote their positions, with $c \in \{x, y, z\}$
- Joint normal distribution for $\mathbf{f}_{t,c}$ and $\mathbf{f}_{t-1,c}$:

$$\begin{bmatrix} \mathbf{f}_{t,c} \\ \mathbf{f}_{t-1,c} \end{bmatrix} \sim \mathcal{N}\left(\mathbf{0}_{2(N+1)}, \begin{bmatrix} \boldsymbol{\Sigma}_{t,t} & \boldsymbol{\Sigma}_{t,t-1} \\ \boldsymbol{\Sigma}_{t,t-1}^T & \boldsymbol{\Sigma}_{t-1,t-1} \end{bmatrix}\right)$$

Derivation of the Manifold-Based Propagation Model (cont.)

- The elements of $\mathbf{\Sigma}_{t,\tau}$ are given by the multiple manifold kernel:

$$\mathrm{cov}(f(\mathbf{h}_r), f(\mathbf{h}_l)) \equiv \tilde{k}(\mathbf{h}_r, \mathbf{h}_l) = \frac{1}{M^2} \sum_{q,w=1}^{M} \sum_{i=1}^{n_L} k_q(\mathbf{h}_r^q, \mathbf{h}_i^q) k_w(\mathbf{h}_l^w, \mathbf{h}_i^w)$$

- The conditional probability is then given by:

$$\mathrm{Pr}\left(\mathbf{f}_{t,c} | \mathbf{f}_{t-1,c}\right) = \mathcal{N}\left(\mathbf{A}_t \mathbf{f}_{t-1,c}, \mathbf{Q}_t\right)$$

where

$$\mathbf{A}_t = \mathbf{\Sigma}_{t,t-1} \mathbf{\Sigma}_{t-1,t-1}^{-1}$$
$$\mathbf{Q}_t = \mathbf{\Sigma}_{t,t} - \mathbf{\Sigma}_{t,t-1} \mathbf{\Sigma}_{t-1,t-1}^{-1} \mathbf{\Sigma}_{t,t-1}^{T}$$

Derivation of the Manifold-Based Propagation Model (cont.)

- The conditional probability induces a linear propagation equation:

$$\mathbf{f}_{t,c} = \mathbf{A}_t \mathbf{f}_{t-1,c} + \boldsymbol{\xi}_t$$

  where $\boldsymbol{\xi}_t \sim \mathcal{N}\left(\mathbf{0}_{N+1}, \mathbf{Q}_t\right)$

- The propagation matrix $\mathbf{A}_t$ and the covariance of the innovation noise $\mathbf{Q}_t$ are time-varying and inferred from the manifold based on the previous and current RTFs and their associated neighbors:

$$\mathbf{h}(t-1), \{\mathbf{h}_{(t-1)_i}\}_{i=1}^{N}, \mathbf{h}(t), \{\mathbf{h}_{t_i}\}_{i=1}^{N}$$

- The position estimate of the test sample $f_c(\mathbf{h}(t))$ is propagated from the previous position estimate, as well as the set of previous neighborhood of the training samples, using the matrices $\mathbf{A}_t$ and $\mathbf{Q}_t$

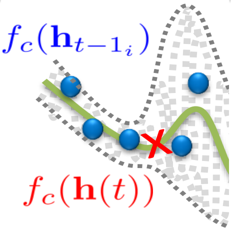
Derivation of the Manifold-Based Propagation Model (cont.)

The full propagation model for the 3-D position

Let $\mathbf{f}_t = \left[\mathbf{f}_{t,x}^T, \mathbf{f}_{t,y}^T, \mathbf{f}_{t,z}^T\right]^T$:

$$\mathbf{f}_t = \mathbf{A}_{3t}\mathbf{f}_{t-1} + \xi_{3t}$$

where $\mathbf{A}_{3t} = \mathbf{A}_t \otimes \mathbf{I}_3$ and $\xi_{3t} \sim \mathcal{N}\left(\mathbf{0}_{3(N+1)}, \mathbf{Q}_{3t}\right)$ with $\mathbf{Q}_{3t} = \mathbf{Q}_t \otimes \mathbf{I}_3$



TDOA-based observation Model

TDOA-based observations:

- Define observations as range differences:

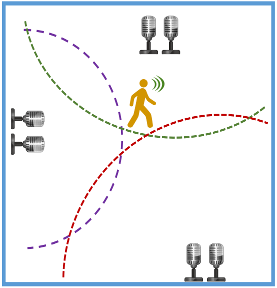
$$\boldsymbol{r} = \left[r^1, \dots, r^M \right]^T$$

- Known nonlinear relation to the source position (requires microphones' positions):

$$r^m = g\left(\mathbf{p}\right) = \left\| \mathbf{p} - \mathbf{q}^{m2} \right\|_2 - \left\| \mathbf{p} - \mathbf{q}^{m1} \right\|_2$$

- The range differences can be extracted from the estimated RTFs [Dvorkind and Gannot, 2005]:

$$\hat{r}^m(t) = \frac{1}{c} \operatorname*{argmax}_{\tau} \hat{h}^m(t,\tau) \equiv \mathrm{IDFT}\left\{ \hat{H}^m(t,k) \right\}$$



TDOA-Based Observation Model

- A nonlinear observation model is formed by:

$$\hat{\mathbf{r}}_t = \mathbf{g}(\mathbf{f}_t) + \zeta_t$$

where $\mathbf{g}(\mathbf{f}_t) = [\mathbf{g}^T(\mathbf{p}(t)), \mathbf{g}^T(\mathbf{p}_{t_1}), \ldots, \mathbf{g}^T(\mathbf{p}_{t_N})]^T$ and

$$\mathbf{g}(\mathbf{p}) = \begin{bmatrix} \left\|\mathbf{p} - \mathbf{q}^{12}\right\|_2 - \left\|\mathbf{p} - \mathbf{q}^{11}\right\|_2 \\ \vdots \\ \left\|\mathbf{p} - \mathbf{q}^{M2}\right\|_2 - \left\|\mathbf{p} - \mathbf{q}^{M1}\right\|_2 \end{bmatrix}$$

and $\zeta_t \sim \mathcal{N}\left(\mathbf{0}_{M(N+1)}, \mathbf{R}_t\right)$ is the observation error

- Linearization of the observation model (Extended Kalman filter - EKF [Smith et al.,]):

$$\nabla_{\mathbf{f}}\mathbf{g}(\mathbf{f}_t) = \mathrm{blkdiag}\left\{\nabla_{\mathbf{p}}\mathbf{g}(\mathbf{p}(t)), \nabla_{\mathbf{p}}\mathbf{g}(\mathbf{p}_{t_1}), \ldots, \nabla_{\mathbf{p}}\mathbf{g}(\mathbf{p}_{t_N})\right\}$$

Tracking Algorithm

Space-state representation:

$$\mathbf{f}_t = \mathbf{A}_{3t}\mathbf{f}_{t-1} + \boldsymbol{\xi}_{3t}$$
$$\hat{\mathbf{r}}_t = \mathbf{g}(\mathbf{f}_t) + \boldsymbol{\zeta}_t$$

EKF: Additional notations

- $\hat{\mathbf{f}}(t|t)$ - The estimate of $\mathbf{f}_t$ based on measurements up to time t
- $\boldsymbol{\Pi}(t|t)$ - The associated error covariance matrix
- $\mathbf{G}_t = \nabla_{\mathbf{f}}\mathbf{g}(\hat{\mathbf{f}}(t|t-1))$ - linearized measurement matrix
- $\mathbf{R}_t$ - Measurement noise (diagonal) covariance matrix, which is significantly lower for the training samples, since their position is known
- $\boldsymbol{\Gamma}(t)$ - Kalman gain

Tracking Algorithm (cont.)

Extended Kalman Filter

Time Update

- Predicted Position:

$$\hat{\mathbf{f}}(t|t-1) = \mathbf{A}_{3t}\hat{\mathbf{f}}(t-1|t-1)$$

- Predicted Covariance:

$$\mathbf{\Pi}(t|t-1) = \mathbf{A}_{3t}\mathbf{\Pi}(t-1|t-1)\mathbf{A}_{3t}^T + \mathbf{Q}_{3t}$$

Measurement Update

- Kalman Gain:

$$\mathbf{\Gamma}(t) = \mathbf{\Pi}(t|t-1)\mathbf{G}_t^T\left(\mathbf{G}_t\mathbf{\Pi}(t|t-1)\mathbf{G}_t^T + \mathbf{R}_t\right)^{-1}$$

- Updated position estimate:

$$\hat{\mathbf{f}}(t|t) = \hat{\mathbf{f}}(t|t-1) + \mathbf{\Gamma}(t)\left(\hat{\mathbf{r}}_t - \mathbf{g}\left(\hat{\mathbf{f}}(t|t-1)\right)\right)$$

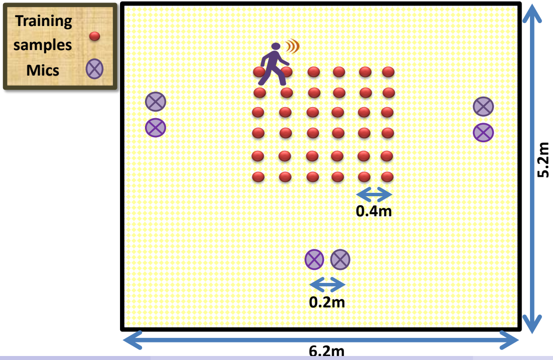
- Updated Covariance:

$$\mathbf{\Pi}(t|t) = \left(\mathbf{I}_{3(N+1)} - \mathbf{\Gamma}(t)\mathbf{G}_t\right)\mathbf{\Pi}(t|t-1)$$

Experimental Results

Setup:

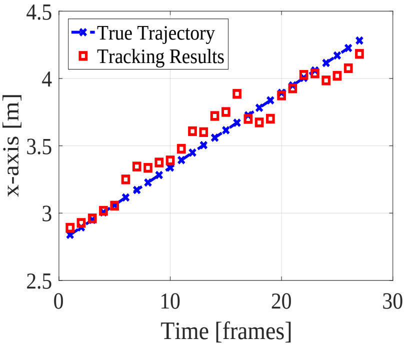
- A $5.2 \times 6.2 \times 3$m room with $T_{60} = 300$ms
- $M = 4$ nodes with 0.2m distance between microphones
- Region of interest: a 2×2m square region
- Training: 36 samples (0.4m resolution)

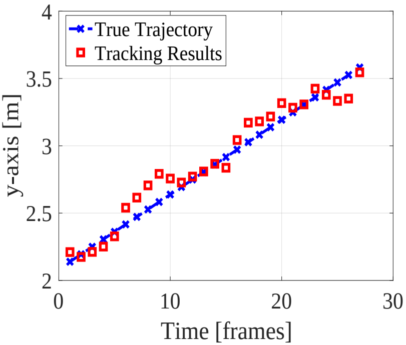


Results

Test I:

- Trajectory: straight line (for 3s)
- Velocity: approximately 1m/s

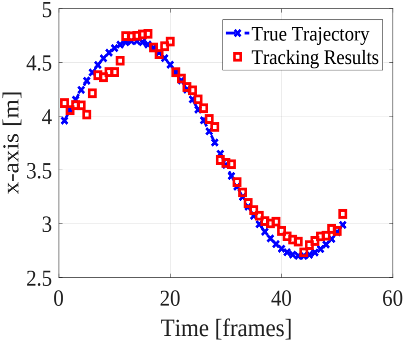


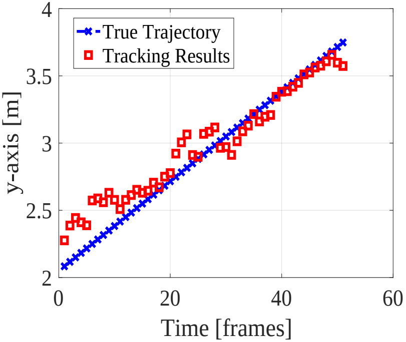


Results

Test II:

- Trajectory: sinusoid (for 5s)
- Velocity: approximately 1m/s

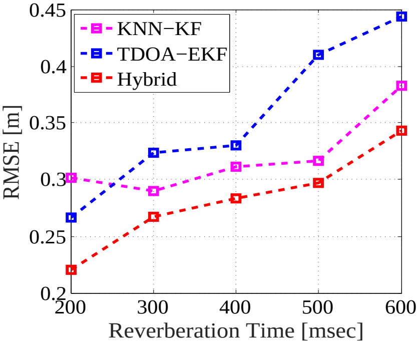


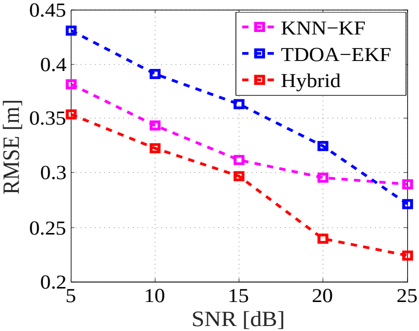


Results

Compare with:

- TDOA-based tracker ('TDOA-EKF') [Gannot and Dvorkind, 2006]: random walk propagation model
- Learning-based approach ('KNN-KF') [Wang and Chaib-Draa, 2013]: linear observation model of labelled positions

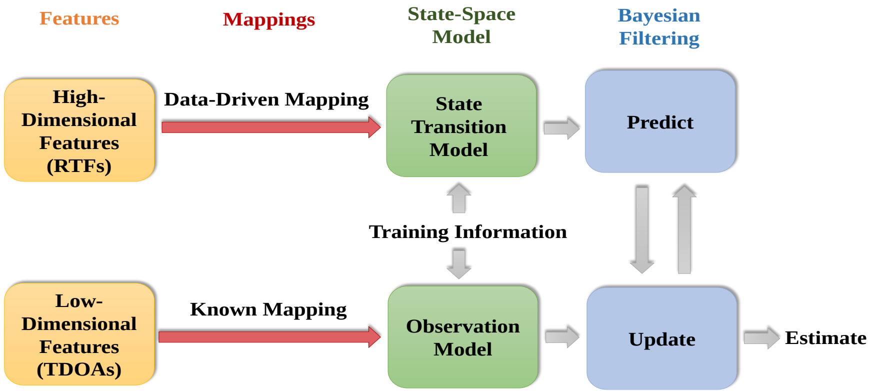




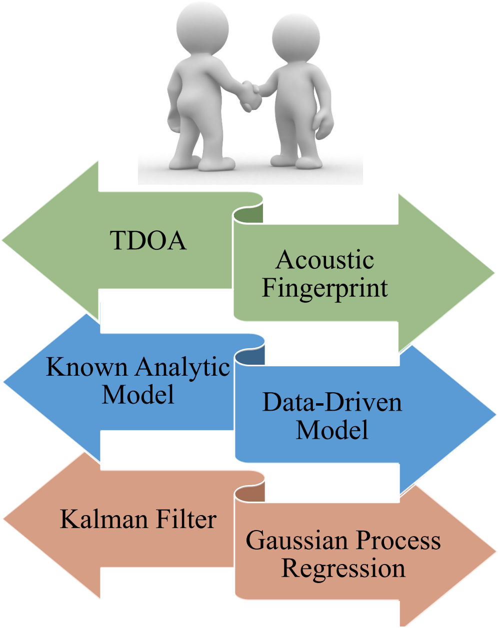
Combining Data Modalities

Combine two data modalities of different types:

- High-dimensional features - data-driven model with acoustic fingerprints
- Low dimensional features - known physical model (TDOA-based)



Classical   Modern



Conclusions

Summary

- Manifold learning approach for source localization
- Data-driven manifold inference
- Location is the controlling variable of the RTF manifold
- Devise algorithms for source localization and tracking using either regularized optimization or Bayesian inference
  - Presents data fusion of several manifolds
  - Dynamics of the source are learned from the variations of the corresponding RTFs on the manifold
- Data-driven, training-based approach, was successfully applied to real-life recordings
- The dynamics on the manifold can be transformed to linear propagation for the source moving in tracking scenarios

Challenges and Perspectives

Challenges

- Robustness to environmental changes:
  - Mismatch between train and test
  - Movements
- Can we apply the approach to multiple concurrent speakers?
- Beamforming is more complicated as it targets enhanced speech rather than its location. Can we extend the approach?
  - A first attempt using projections to the inferred manifold [Talmon and Gannot, 2013]

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