Introduction to distributed speech enhancement algorithms for ad hoc microphone arrays and wireless acoustic sensor networks

Part IV: Random Microphone Deployment

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Shmulik Markovich-Golan

Blind Sampling Rate Offset Estimation and Compensation

[Markovich-Golan et al., 2012]

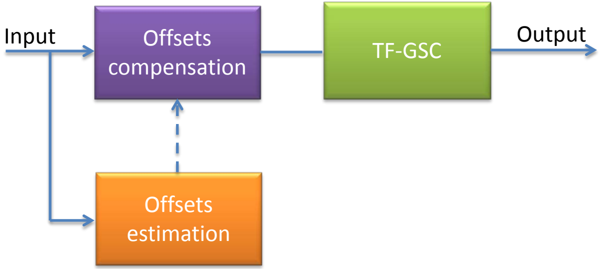
Scenario

- Fully connected N nodes network with M_n microphones at the nth node.
- Nominal sampling rate f_s.
- Sampling rate $f_{s,n} = (1 + \epsilon_n)\, f_s$, sampling period $T_{s,n}$ with Sampling rate offset ϵ_n.

TF-GSC [Gannot et al., 2001] with Sampling Rate Offsets

- RTF is constantly changing: signal distortion.
- ANC is constantly updating: increased noise level.
- Microphone signals are less coherent: degraded performance.

Block Diagram of Synchronized TF-GSC

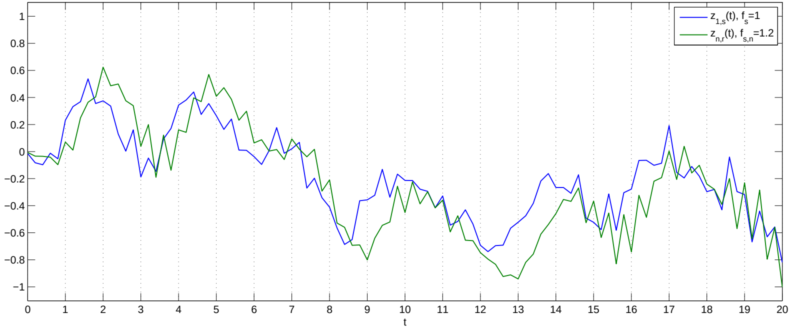


Synchronized TF-GSC

- Sampling rate estimation: based on the phase drift of the coherence between microphones in stationary noise-only segments (in coherent frequency bands).
- Resampling with Lagrange polynomials interpolation [Erup et al., 1993].
- Other beamforming sync. methods: [Wehr et al., 2004]; [Ono et al., 2009]; [Liu, 2008].

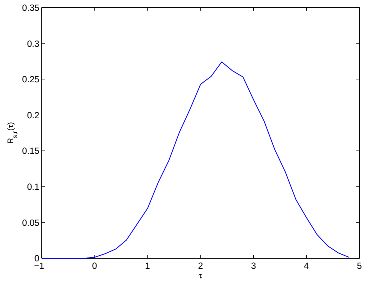
Continuous Microphone Signals

- Received noise component at microphone s, 1st node: $v_{1,s}(t)$.
- Received noise component at microphone r, nth node: $v_{n,r}(t)$.
- Noise only time-segment.

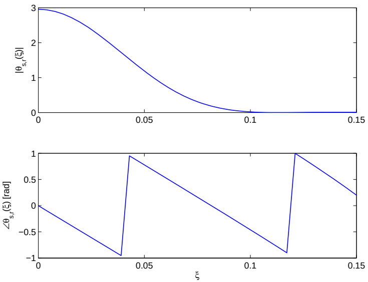


Statistics of Noise Components $v_{1,s}(t)$ and $v_{n,r}(t)$

- Cross-covariance: $R_{s,r}(\tau) = \mathrm{E}\{v_{1,s}(t)\, v_{n,r}(t-\tau)\}$.
- Cross-spectrum: $\theta_{s,r}(\xi) = \int_{-\infty}^{\infty} R_{s,r}(\tau) \exp(-\mathrm{j}\xi\tau)\, \mathrm{d}\tau$.



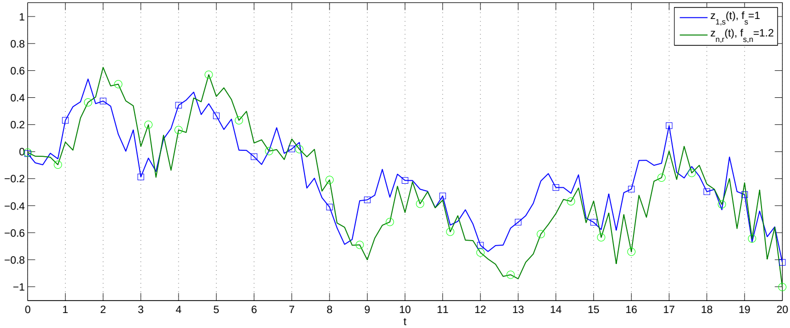
Cross-covariance



Cross-spectrum

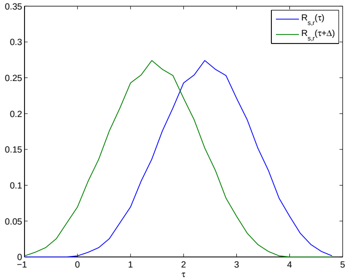
Sampled Microphone Signals

- $v_{1,s}[\ell] = v_{1,s}(\ell T_s)$.
- $v_{n,r}[\ell] = v_{n,r}(\ell T_{s,n})$.

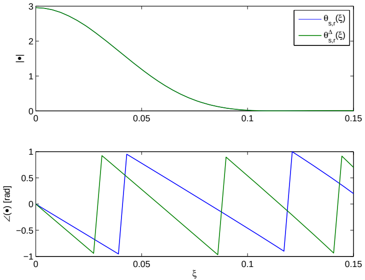


Statistics of Microphones $v_{1,s}(t)$ and $v_{n,r}(t-\Delta)$

- Cross-covariance: $R_{s,r}(\tau+\Delta)$.
- Cross-spectrum: $\theta^{\Delta}_{s,r}(\xi) = \exp(\mathrm{j}\xi\Delta)\,\theta_{s,r}(\xi)$.
- Time difference at the ℓth sample: $\Delta = \ell T_s - \ell T_{s,n} \approx \ell T_s \epsilon_n$ (using first-order Taylor series approximation).



Cross-covariance



Cross-spectrum

Statistics of Sampled Microphones $v_{1,s}[\ell]$ and $v_{n,r}[\ell]$

Cross-spectrum is band-limited by $\frac{f_s}{2}$

- Assume small offset.
- $\theta^{\ell}_{s,r}[k] = \theta^{\ell T_s \epsilon_n}_{s,r}\left(\frac{2\pi k f_s}{K}\right)$.
- Let, $\theta_{s,r}[k] = \theta_{s,r}\left(\frac{2\pi k f_s}{K}\right)$, then:

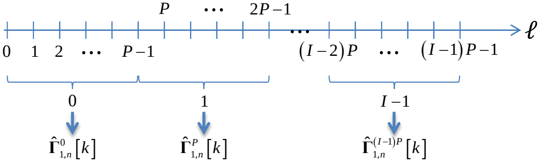
$$\theta^{\ell}_{s,r}[k] = \exp\left(\mathrm{j}\frac{2\pi k \ell \epsilon_n}{K}\right)\theta_{s,r}[k]$$

Coherence between microphones s and r at the ℓth sample

- Define $\gamma^{\ell}_{s,r}[k] = \frac{\theta^{\ell}_{s,r}[k]}{\sqrt{\theta_{s,s}[k]\theta_{r,r}[k]}}$ and $\gamma_{s,r}[k] = \frac{\theta_{s,r}[k]}{\sqrt{\theta_{s,s}[k]\theta_{r,r}[k]}}$; for $k = 0, 1, \ldots, K-1$.
- Then $\gamma^{\ell}_{s,r}[k] = \alpha^{\ell}_n \gamma_{s,r}[k]$ with $\alpha_n = \exp\left(\mathrm{j}\frac{2\pi k \epsilon_n}{K}\right)$.
- ϵ_n can be extracted from α_n.

Offset Estimation at the *n*th Node

- Given a noise-only time segment of P_s samples.
- Partition into I frames of P samples: $\ell = i \times P;\ i = 0, 1, \ldots, I-1$.
- estimate $M_1 \times M_n$ coherence matrix $\hat{\boldsymbol{\Gamma}}_{1,n}^{iP}[k]$ between microphones of the 1st and the *n*th nodes.



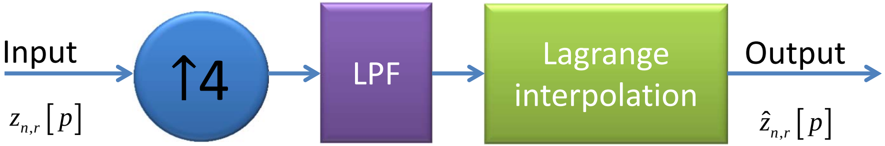
- $|\epsilon_n| < \epsilon_{\max} \Rightarrow$ no 2π ambiguity for $k \leq k_{\max} = \frac{K}{2P\epsilon_{\max}}$.
- Estimate the *n*th node sampling rate offset:
  - s, r pair: $\hat{\epsilon}_{n,s,r} = \text{avg}_k \left(\frac{K}{2\pi P k} \angle \underbrace{\underset{i}{\text{avg}}\, \hat{\gamma}_{s,r}^{iP}[k] / \gamma_{s,r}^{(i-1)P}[k]}_{\hat{\alpha}_n^P} \right)$.
  - Average all microphone pairs: $\hat{\epsilon}_n = \frac{1}{M_1 M_n} \sum_{s=1}^{M_1} \sum_{r=1}^{M_n} \hat{\epsilon}_{n,s,r}$.

Resampling with Lagrange Polynomials Interpolation

[Pawig et al., 2010],[Erup et al., 1993]

Resample $z_{n,r}(pT_{s,n})$ to $z_{n,r}(pT_s)$

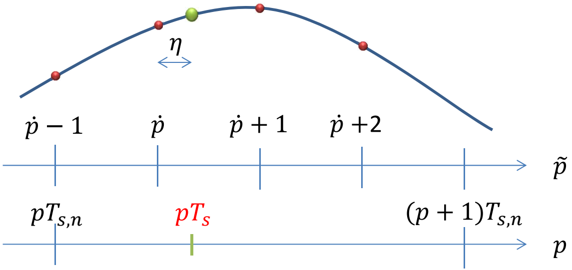
- Interpolate $z_{n,r}[p]$ by factor 4: $\tilde{z}_{n,r}[\tilde{p}]$.
- Denote: $\dot{p} = \lfloor \frac{4pT_s}{T_{s,n}} \rfloor = \lfloor 4p(1+\hat{\epsilon}_n) \rfloor$, the closest interpolated sample index from the left to time pT_s.
- The resampled value of $z_{n,r}(pT_s)$ is $\hat{z}_{n,r}[p] = \beta^p_{-1}\tilde{z}_{n,r}[\dot{p}-1] + \beta^p_0\tilde{z}_{n,r}[\dot{p}] + \beta^p_1\tilde{z}_{n,r}[\dot{p}+1] + \beta^p_2\tilde{z}_{n,r}[\dot{p}+2]$.



Resampling with Lagrange Polynomials Interpolation [Pawig et al., 2010],[Erup et al., 1993] II

Calculate Weights

- $\eta = 4p\left(1 + \hat{\epsilon}_n\right) - \dot{p}$.
- $\beta^p_{-1} = -\frac{\eta(\eta-1)(\eta-2)}{6}$.
- $\beta^p_0 = \frac{(\eta+1)(\eta-1)(\eta-2)}{2}$.
- $\beta^p_1 = -\frac{(\eta+1)\eta(\eta-2)}{2}$.
- $\beta^p_2 = \frac{(\eta+1)\eta(\eta-1)}{6}$.



Experimental Study

Q directional stationary interfering sources

TF-GSC Algorithms

W.o. offsets; Conventional TF-GSC; Synchronized TF-GSC

Criteria

Signal to Distortion ratio (SDR); Signal to Noise (SNR)

	Without offset		With offset			
	Conventional		Conventional		Synchronized	
Q	SDR	SNR	Ex. Dist.	Ex. Noise	Ex. Dist.	Ex. Noise
1	15.0	34.3	11.2	7.7	0.0	0.0
2	14.9	27.5	11.2	4.9	0.1	0.0
3	14.6	24.5	11.5	3.4	0.4	0.1
4	14.7	23.5	11.9	2.9	0.8	0.2

Values in dB, Ex. - excess values

WASNs with Random Node Deployment

[Markovich-Golan et al., 2011]; [Markovich-Golan et al., 2013]; general reading [Lo, 1964]

Scenarios

- Ad hoc sensor networks.
- Large volume (and many microphones).
- High fault percentage.
- Arbitrary microphone deployment.

Questions

- How many microphones are required?
- What is the expected performance?
- Is there an optimal deployment? [Kodrasi et al., 2011]

Outline

- Array of randomly located microphones in a reverberant enclosure.
- Single desired speaker.
- Utilizing the statistical model of the ATFs, statistical models for the SIR and WNG are derived.
- The reliability of the SDW-MWF is computed for:
  - Multiple coherent noise sources.
  - Diffuse sound field.
- The reliability functions can be used to determine the number of microphones required to assure a desired performance level (with a controlled level of uncertainty).

Notations I

Signals

- Let $s_d(\ell, k)$ be a desired speaker signal located at $\mathbf{r}_d$.
- Microphone signals:

$$\mathbf{z}(\ell, k) \triangleq \mathbf{h}_d(\ell, k)\, s_d(\ell, k) + \mathbf{v}(\ell, k).$$

- Microphone signals PSD:

$$\mathbf{\Phi}_{zz}(\ell, k) \triangleq \mathrm{E}\{\mathbf{z}(\ell, k)\mathbf{z}^H(\ell, k)\} = \sigma_d^2(\ell, k)\mathbf{h}_d(\ell, k)\mathbf{h}_d^H(\ell, k) + \mathbf{\Phi}_{vv}(\ell, k)$$

- Noise PSD:

$$\mathbf{\Phi}_{vv}(\ell, k) \triangleq \mathrm{E}\{\mathbf{v}(\ell, k)\mathbf{v}^H(\ell, k)\}.$$

Notations II

Room Constellation

- Room volume and surface area:

$$V \triangleq D_x \times D_y \times D_z$$
$$A \triangleq 2\left(D_x \times D_y + D_x \times D_z + D_y \times D_z\right)$$

- Reverberation time: T_{60}.
- M microphones randomly deployed with a uniform distribution at coordinates $\mathbf{r}^m \triangleq \left[\, r_x^m \;\; r_y^m \;\; r_z^m \,\right]^T$; $m = 1, \ldots, M$.

Criterion

SDW-MWF

$$\mathbf{w} \triangleq \underset{\mathbf{w}'}{\operatorname{argmin}} \left|1 - \left(\left(\mathbf{w}'\right)^H \mathbf{h}_d\right)\right|^2 \sigma_d^2 + \mu \left(\mathbf{w}'\right)^H \mathbf{\Phi}_{vv} \mathbf{w}' = \frac{\mathbf{\Phi}_{vv}^{-1} \mathbf{h}_d}{\mathbf{h}_d^H \mathbf{\Phi}_{vv}^{-1} \mathbf{h}_d + \frac{\mu}{\sigma_d^2}}$$

SINR and WNG are Random Variables

Signal to Interference and Noise (SINR):

$$\kappa \triangleq \frac{\sigma_d^2 |\mathbf{w}^H \mathbf{h}_d|^2}{\mathbf{w}^H \mathbf{\Phi}_{vv} \mathbf{w}} = \sigma_d^2 \mathbf{h}_d^H \mathbf{\Phi}_{vv}^{-1} \mathbf{h}_d$$

White noise gain (WNG):

$$\xi \triangleq \frac{|\mathbf{w}^H \mathbf{h}_d|^2}{\|\mathbf{w}\|^2} = \frac{\left(\mathbf{h}_d^H \mathbf{\Phi}_{vv}^{-1} \mathbf{h}_d\right)^2}{\mathbf{h}_d^H \mathbf{\Phi}_{vv}^{-2} \mathbf{h}_d}$$

Statistical ATF Modelling

ATF relating a coherent source at $\mathbf{r}_d$, and the mth microphone at $\mathbf{r}^m$

$$h \triangleq \bar{h} + \hat{h}$$

- $\bar{h}$ the direct arrival.
- $\hat{h}$ the reverberant component.
- The direct arrival and the reverberant tail assumed uncorrelated.

Reverberant Tail Model [Schroeder, 1987],[Kuttruff, 2000]

Under the Assumptions:

- The signal wavelength is much smaller than the room dimensions.
- The microphones and sources are at least half wavelength away from the walls.
- The signal frequency is above the Schroeder frequency, $f_{\mathrm{Schroeder}} \triangleq 2000\sqrt{\frac{T_{60}}{V}}$ (typically few hundred Hz).

The Tail Statistics:

$$\hat{h} \sim \mathcal{CN}\left(0, \hat{\alpha}\right)$$

with $\hat{\alpha} \triangleq \frac{1-\varepsilon}{\pi\varepsilon A}$ and $\varepsilon \triangleq \frac{0.161V}{AT_{60}}$, the exponential decay rate of the RIR tail.

The Direct Arrival (Spherical Wave Propagation)

The Direct Arrival Model

$$\bar{h} \triangleq \bar{a} \exp\left(j\bar{\phi}\right)$$

where:

$$\bar{a} = \begin{cases} 1 & ; \mathbf{r}_d \leq \frac{1}{4\pi} \\ \frac{1}{4\pi\|\mathbf{r}_d - \mathbf{r}^m\|} & ; \frac{1}{4\pi} < \mathbf{r}_d \end{cases}$$

$$\bar{\phi} = \frac{2\pi\|\mathbf{r}_d - \mathbf{r}^m\|}{\lambda_k}$$

Second-Order Statistics of single ATF

Arbitrary Sensor Location within $\bar{r}$

Under the Assumptions:

- For $\bar{r} \gg r_c$, where $r_c \triangleq \sqrt{\frac{V}{100\pi T_{60}}}$ is the critical distance, the direct path is negligible [Kuttruff, 2000].
- For $\bar{r} \gg \lambda_k$ multiple 2π phase cycles are repeated while sound wave propagates.
- $\bar{r}$ arbitrarily chosen (the results are not sensitive to the exact value).

Approximations:

- $\mathrm{E}\left\{\bar{h}\right\} \approx 0 \Rightarrow \mathrm{E}\left\{h\right\} = 0$.
- $\mathrm{E}\left\{|h|^2\right\} \triangleq \alpha = \frac{4\pi\bar{r}^3}{3V}\bar{\alpha} + \hat{\alpha}$ with $\bar{\alpha} = \frac{6\pi\bar{r}-1}{32\pi^3\bar{r}^3}$.
- The ATFs $h_m;\ m = 1,\ dots, M$ relating the source and randomly deployed microphones are i.i.d. (for large sphere and few microphones).

Covariance of ATFs Relating 2 Sources and Randomly Located Microphone

ATFs covariance:

$$\mathrm{E}\left\{h_1 h_2^*\right\} = \mathrm{E}\left\{\bar{h}_1 \bar{h}_2^*\right\} + \mathrm{E}\left\{\hat{h}_1 \hat{h}_2^*\right\}$$

- Reverberant tail is diffused [Jacobsen and Roisin, 2000]:

$$\mathrm{E}\left\{\hat{h}_1 \hat{h}_2^*\right\} = \hat{\alpha}\,\mathrm{sinc}\left(\frac{2\pi\|\mathbf{r}_1 - \mathbf{r}_2\|}{\lambda_k}\right)$$

- Assuming $\|\mathbf{r}_1 - \mathbf{r}_2\| \gg \lambda_k$:
  - $\mathrm{E}\left\{\hat{h}_1 \hat{h}_2^*\right\} \approx 0$, since the sinc is decaying.
  - $\mathrm{E}\left\{\bar{h}_1 \bar{h}_2^*\right\} \approx 0$, since multiple 2π phase cycles are repeated while sound wave propagates.

Reliability Measures

SIR Reliability

The reliability of an SIR level of κ_0 is defined as the probability that the output SIR will exceed κ_0:

$$R_\kappa(\kappa_0) \triangleq \Pr(\kappa \geq \kappa_0).$$

White Noise Gain (WNG) Reliability

The reliability of a WNG level of ξ_0 is defined as the probability that the WNG will exceed ξ_0:

$$R_\xi(\xi_0) \triangleq \Pr(\xi \geq \xi_0).$$

P Directional Noise Sources

For high INR and $P \ll M$

$$R_{\kappa,c}(\kappa_0) = 1 - F_{\eta,c}\left(\frac{2}{\alpha}\frac{\sigma_u^2}{\sigma_d^2}\kappa_0\right)$$
$$R_{\xi,c}(\xi_0) = 1 - F_{\eta,c}\left(\frac{2}{\alpha}\xi_0\right).$$

- σ_u^2 - sensor noise variance and σ_d^2 - desired source varaince.
- $\eta_c \sim \chi^2(2(M-P))$ Chi-square RV with $2(M-P)$ degrees of freedom.
- $F_{\eta,c}(\eta_0) \triangleq \Pr(\eta_c \leq \eta_0) = \frac{\gamma_f\left(M-P,\frac{\eta_0}{2}\right)}{\Gamma_f(M-P)}$ is the respective CDF.
- Γ_f is the Gamma function.
- γ_f is the lower incomplete Gamma function.

Diffused Noise Source

Noise Field

$$\Phi_{vv}\left(m, m'\right) = \sigma_{\text{dif}}^2 \operatorname{sinc}\left(\frac{2\pi \|\mathbf{r}_m - \mathbf{r}_{m'}\|}{\lambda_k}\right) \approx \sigma_{\text{dif}}^2 \mathbf{I}.$$

- σ_{dif}^2 variance of the diffuse field.
- For enclosures larger than λ_k.

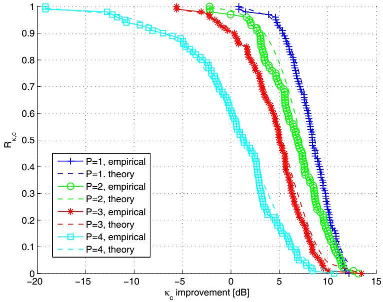
Reliability

$$R_{\kappa,\text{dif}}\left(\kappa_0\right) = 1 - F_{\eta,\text{dif}}\left(\frac{2}{\alpha}\frac{\sigma_{\text{dif}}^2}{\sigma_d^2}\kappa_0\right)$$

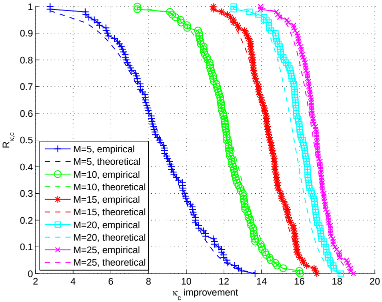
$$R_{\xi,\text{dif}}\left(\xi_0\right) = 1 - F_{\eta,\text{dif}}\left(\frac{2}{\alpha}\xi_0\right)$$

- $\eta_{\text{dif}} \sim \chi^2\left(2M\right)$ Chi-square RV with $2M$ degrees of freedom.
- $F_{\eta,\text{dif}}\left(\eta_0\right) \triangleq \Pr\left(\eta_{\text{dif}} \leq \eta_0\right) = \frac{\gamma_f\left(M, \frac{\eta_0}{2}\right)}{\Gamma_f(M)}$ is the respective CDF.

SIR Reliability



(a) $M = 5$



(b) $P = 1$

$\mathrm{SINR}_{\mathrm{out}} - \mathrm{SNR}_{\mathrm{in}}$ for coherent noise sources. $T_{60} = 0.4$sec, room dimensions $4 \times 4 \times 3$m. Similar trends for diffused noise field.

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